The proof of a conjecture concerning the intersection of k-generalized Fibonacci sequences

Diego Marques[1]

Departamento de Matemática, Universidade de Brasília, Brasília, 70910-900, Brazil

Abstract

For $k \geq 2$, the k-generalized Fibonacci sequence $(F_n^{(k)})_n$ is defined by the initial values $0, 0, ..., 0, 1$ (k terms) and such that each term afterwards is the sum of the k preceding terms. In 2005, Noe and Post conjectured that the only solutions of Diophantine equation $F_m^{(k)} = F_n^{(\ell)}$, with $\ell > k > 1, n > \ell + 1,\ m > k + 1$ are

$$(m, n, \ell, k) = (7, 6, 3, 2) \text{ and } (12, 11, 7, 3).$$

In this paper, we confirm this conjecture.

Keywords: k-generalized Fibonacci numbers, linear forms in logarithms, intersection
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1. Introduction

Let $k \geq 2$ and denote $F^{(k)} := (F_n^{(k)})_{n \geq -(k-2)}$, the *$k$-generalized Fibonacci sequence* whose terms satisfy the recurrence relation

$$F_{n+k}^{(k)} = F_{n+k-1}^{(k)} + F_{n+k-2}^{(k)} + \cdots + F_n^{(k)}, \tag{1}$$

with initial conditions $0, 0, ..., 0, 1$ (k terms) and such that the first nonzero term is $F_1^{(k)} = 1$.

Email address: `diego@mat.unb.br` (Diego Marques)
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The above sequence is one among the several generalizations of Fibonacci numbers. Such a sequence is also called *k-step Fibonacci sequence*, the *Fibonacci k-sequence*, or *k-bonacci sequence*. Clearly for $k = 2$, we obtain the well-known Fibonacci numbers $F_n^{(2)} = F_n$, and for $k = 3$, the Tribonacci numbers $F_n^{(3)} = T_n$.

Several authors have worked on problems involving k-generalized Fibonacci sequences. For instance, Togbé and the author [13] proved that only finitely many terms of a linear recurrence sequence whose characteristic polynomial has a simple positive dominant root can be *repdigits* (i.e., numbers with only one distinct digit in its decimal expansion). As an application, since the characteristic polynomial of the recurrence in (1), namely $x^k - x^{k-1} - \cdots - x - 1$, has just one root α such that $|\alpha| > 1$ (see for instance [24]), then there exist only finitely many terms of $F^{(k)}$ which are repdigits, for all $k \geq 2$. F. Luca [12] and the author [15] proved that 55 and 44 are the largest repdigits in the sequences $F^{(2)}$ and $F^{(3)}$, respectively. Moreover, the author conjectured that there are no repdigits, with at least two digits, belonging to $F^{(k)}$, for $k > 3$. In a recent work, Bravo and Luca [3] confirmed this conjecture.

Here, we are interested in the problem of determining the intersection of two k-generalized Fibonacci sequences. It is important to notice that Mignotte (see [17]) showed that if $(u_n)_{n\geq 0}$ and $(v_n)_{n\geq 0}$ are two linearly recurrence sequences then, under some weak technical assumptions, the equation

$$u_n = v_m$$

has only finitely many solutions in positive integers m, n. Moreover, all such solutions are effectively computable (we refer the reader to [1, 20, 21, 23] for results on the intersection of two recurrence sequences). Thus, it is reasonable to think that the intersection $F^{(k)} \cap F^{(\ell)}$ is a finite set for all $2 \leq k < \ell$. In 2005, Noe and Post [18] gave a heuristic argument to show that the expected cardinality of this intersection must be small. Furthermore, they raised the following conjecture

Conjecture 1 (Noe-Post). *The Diophantine equation*

$$F_m^{(k)} = F_n^{(\ell)}, \tag{2}$$

with $\ell > k \geq 2$, $n > \ell + 1$ *and* $m > k + 1$, *has only the solutions:*

$$(m, n, \ell, k) = (7, 6, 3, 2) \text{ and } (12, 11, 7, 3). \tag{3}$$

That is,

$$13 = F_7^{(2)} = F_6^{(3)} \quad and \quad 504 = F_{12}^{(3)} = F_{11}^{(7)}$$

Since the first nonzero terms of $F^{(k)}$ are $1, 1, 2, ..., 2^{k-1}$, then the above conjecture can be rephrased as

Conjecture 2. *Let $2 \leq k < \ell$ be positive integer numbers. Then*

$$F^{(k)} \cap F^{(\ell)} = \begin{cases} \{0,1,2,13\}, & if \quad (k,\ell) = (2,3) \\ \{0,1,2,4,504\}, & if \quad (k,\ell) = (3,7) \\ \{0,1,2,8\}, & if \quad k = 2 \text{ and } \ell > 3 \\ \{0,1,2,\ldots,2^{k-1}\}, & otherwise \end{cases}$$

We remark that this intersection was confirmed for $(k,\ell) = (2,3)$, by the author [14]. Also, Noe and Post used computational methods to study this intersection (see Section 5). Let us state their result as a lemma, since we shall use it throughout our work.

Lemma 1. *The only solutions (m,n,ℓ,k) in positive integers of Diophantine equation (2), with $\ell > k > 1, n > \ell + 1,\ m > k + 1$ and $\max\{m,n,k,\ell\} < 2^{2000}$, are listed in (3).*

In this paper, we shall use transcendental tools to prove the Noe-Post conjecture. For the sake of preciseness, we stated it as a theorem.

Theorem 1. *Conjecture 1 is true.*

Let us give a brief overview of our strategy for proving Theorem 1. First, we use a Dresden formula [6, Formula (2)] to get an upper bound for a linear form in three logarithms related to equation (2). After, we use a lower bound due to Matveev to obtain an upper bound for m and n in terms of ℓ. Very recently, Bravo and Luca solved the equation $F_n^{(k)} = 2^m$ and for that they used a nice argument combining some estimates together with the Mean Value Theorem (this can be seen in pages 72 and 73 of [2]). In our case, we must use two times this Bravo and Luca approach to prove our main theorem. In the final section, we present a program for checking the "small" cases. The computations in the paper were performed using *Mathematica*®.

We remark some differences between our work and the one by Bravo and Luca. In their paper, the equation $F_n^{(k)} = 2^m$ was studied. By applying a key method, they get directly an upper bound for $|2^m - 2^{n-2}|$. In our case, the equation $F_m^{(k)} = F_n^{(\ell)}$ needs a little more work, because it is necessary to apply two times their method to get an upper bound for $|2^{n-2} - 2^{m-2}|$. Moreover, they used a reduction argument due to Dujella and Pethö to solve small cases. In our work, we use a Noe and Post program to deal with these cases. Our presentation is therefore organized in a similar way that the one in the papers [2, 3], since we think that those presentations are intuitively clear.

2. Upper bounds for m and n in terms of ℓ

In this section, we shall prove the following result

Lemma 2. *If (m, n, ℓ, k) is a solution in positive integers of Diophantine equation (2), with $\ell > k \geq 2$, $n > \ell + 1$ and $m > k + 1$. Then*

$$n < m < 4.4 \cdot 10^{14} \ell^8 \log^3 \ell.$$

Before proceeding further, we shall recall some facts and properties of these sequences which will be used after.

We know that the characteristic polynomial of $(F_n^{(k)})_n$ is

$$\psi_k(x) := x^k - x^{k-1} - \cdots - x - 1$$

and it is irreducible over $\mathbb{Q}[x]$ with just one zero outside the unit circle. That single zero is located between $2(1-2^{-k})$ and 2 (as can be seen in [24]). Also, in a recent paper, G. Dresden [6, Theorem 1] gave a simplified "Binet-like" formula for $F_n^{(k)}$:

$$F_n^{(k)} = \sum_{i=1}^{k} \frac{\alpha_i - 1}{2 + (k+1)(\alpha_i - 2)} \alpha_i^{n-1}, \tag{4}$$

for $\alpha = \alpha_1, ..., \alpha_k$ being the roots of $\psi_k(x)$. There are many other ways of representing these k-generalized Fibonacci numbers, as can be seen in [7, 8, 9, 10]. Also, it was proved in [3, Lemma 1] that

$$\alpha^{n-2} \leq F_n^{(k)} \leq \alpha^{n-1}, \text{ for all } n \geq 1, \tag{5}$$

where α is the dominant root of $\psi_k(x)$. Also, the contribution of the roots inside the unit circle in formula (4) is almost trivial. More precisely, it was proved in [6] that

$$|F_n^{(k)} - g(\alpha, k)\alpha^{n-1}| < \frac{1}{2}, \tag{6}$$

where we adopt throughout the notation $g(x,y) := (x-1)/(2+(y+1)(x-2))$.

As a last tool to prove Lemma 2, we still use a lower bound for a linear form logarithms *à la Baker* and such a bound was given by the following result of Matveev (see [16] or Theorem 9.4 in [4]).

Lemma 3. *Let $\gamma_1, \ldots, \gamma_t$ be real algebraic numbers and let $b_1, \ldots, b_t$ be nonzero rational integer numbers. Let D be the degree of the number field $\mathbb{Q}(\gamma_1, \ldots, \gamma_t)$ over $\mathbb{Q}$ and let A_j be a positive real number satisfying*

$$A_j \geq \max\{Dh(\gamma_j), |\log \gamma_j|, 0.16\} \text{ for } j = 1, \ldots, t.$$

Assume that

$$B \geq \max\{|b_1|, \ldots, |b_t|\}.$$

If $\gamma_1^{b_1} \cdots \gamma_t^{b_t} \neq 1$, then

$$|\gamma_1^{b_1} \cdots \gamma_t^{b_t} - 1| \geq \exp(-1.4 \cdot 30^{t+3} \cdot t^{4.5} \cdot D^2(1 + \log D)(1 + \log B)A_1 \cdots A_t).$$

As usual, in the above statement, the *logarithmic height* of an s-degree algebraic number γ is defined as

$$h(\gamma) = \frac{1}{s}(\log |a| + \sum_{j=1}^{s} \log \max\{1, |\gamma^{(j)}|\}),$$

where a is the leading coefficient of the minimal polynomial of γ (over $\mathbb{Z}$) and $(\gamma^{(j)})_{1 \leq j \leq s}$ are the conjugates of γ (over $\mathbb{Q}$).

2.1. The proof of Lemma 2

First, the inequality $n < m$ follows from the facts that the sequences $(F_n^{(\ell)})_n$ and $(F_n^{(\ell)})_\ell$ are nondecreasing together with (2), $n > \ell + 1$ and $m > k + 1$. By the way, to find an upper bound for m in terms of n, we combine (2) and (5) to obtain

$$2^{n-1} > \phi^{n-1} \geq F_n^{(\ell)} = F_m^{(k)} \geq \alpha^{m-2} > (\sqrt{2})^{m-2} \text{ and so } 2n > m, \qquad (7)$$

where in the last inequality we used that $\alpha > 3/2 > \sqrt{2}$.

Now, we use (6) to get

$$|F_m^{(k)} - g(\alpha, k)\alpha^{m-1}| < \frac{1}{2} \text{ and } |F_n^{(\ell)} - g(\phi, \ell)\phi^{n-1}| < \frac{1}{2},$$

where α and ϕ are the dominant roots of the recurrences $(F_m^{(k)})_m$ and $(F_n^{(\ell)})_n$, respectively. Combining these inequalities, we obtain

$$|g(\phi,\ell)\phi^{n-1} - g(\alpha,k)\alpha^{m-1}| < 1 \tag{8}$$

and so

$$\left|\frac{g(\phi,\ell)\phi^{n-1}}{g(\alpha,k)\alpha^{m-1}} - 1\right| < \frac{1}{g(\alpha,k)\alpha^{m-1}} < \frac{4}{\alpha^{m-1}}, \tag{9}$$

where we used that $g(\alpha,k) > 1/4$, since $\alpha > 3/2$ (for $k \geq 2$) and $2 + (k+1)(\alpha-2) < 2$. Thus (9) becomes

$$|e^{\Lambda} - 1| < \frac{4}{\alpha^{m-1}}, \tag{10}$$

where $\Lambda := (n-1)\log\phi + \log(g(\phi,\ell)/g(\alpha,k)) - (m-1)\log\alpha$.

Now, we shall apply Lemma 3. To this end, take $t := 3$,

$$\gamma_1 := \phi,\ \gamma_2 := \frac{g(\phi,\ell)}{g(\alpha,k)},\ \gamma_3 := \alpha$$

and

$$b_1 := n-1,\ b_2 := 1,\ b_3 := m-1.$$

For this choice, we have $D = [\mathbb{Q}(\alpha,\phi) : \mathbb{Q}] \leq k\ell < \ell^2$. Also $h(\gamma_1) = (\log\phi)/\ell < (\log 2)/\ell < 0.7/\ell$ and similarly $h(\gamma_3) < 0.7/k$. In [2, p. 73], an estimate for $h(g(\alpha,k))$ was given. More precisely, it was proved that

$$h(g(\alpha,k)) < \log(k+1) + \log 4.$$

Analogously,

$$h(g(\phi,\ell)) < \log(\ell+1) + \log 4.$$

Thus

$$h(\gamma_2) \leq h(g(\phi,\ell)) + h(g(\alpha,k)) \leq \log(\ell+1) + \log(k+1) + 2\log 4,$$

where we used the well-known facts that $h(xy) \leq h(x) + h(y)$ and $h(x) = h(x^{-1})$. Also, in [2] was proved that $|g(\alpha_i,k)| < 2$, for all $i = 1,\ldots,k$.

Since $\ell > k$ and $m > n$, we can take $A_1 = A_3 := 0.7\ell$, $A_2 := 2\ell^2\log(4\ell+4)$ and $B := m-1$.

Before applying Lemma 3, it remains us to prove that $e^{\Lambda} \neq 1$. Suppose, towards a contradiction, the contrary, i.e., $g(\alpha,k)\alpha^{m-1} = g(\phi,\ell)\phi^{n-1} \in \mathbb{Q}(\phi)$. So, we can conjugate this relation in $\mathbb{Q}(\phi)$ to get

$$g(\alpha_{s_i},k)\alpha_{s_i}^{m-1}=g(\phi_i,\ell)\phi_i^{n-1},\text{ for } i=1,\ldots,\ell,$$

where α_{s_i} are the ℓ conjugates of α over $\mathbb{Q}(\phi)$. Since $g(\alpha,k)\alpha^{m-1}$ has at most k conjugates (over $\mathbb{Q}$), then each number in the list $\{g(\alpha_{s_i},k)\alpha_{s_i}^{m-1}:1\le i\le \ell\}$ is repeated at least $\ell/k>1$ times. In particular, there exists $t\in\{2,\ldots,\ell\}$, such that $g(\alpha_{s_1},k)\alpha_{s_1}^{m-1}=g(\alpha_{s_t},k)\alpha_{s_t}^{m-1}$. Thus, $g(\phi,k)\phi^{n-1}=g(\phi_t,\ell)\phi_t^{n-1}$ and then

$$\left(\frac{7}{4}\right)^{n-1}<\phi^{n-1}=\left|\frac{g(\phi_t,\ell)}{g(\phi,\ell)}\right||\phi_t|^{n-1}<8,$$

where we used that $\phi>2(1-2^{-\ell})\ge 7/4$, $|g(\phi_t,\ell)|<2<8|g(\phi,\ell)|$ and $|\phi_t|<1$ for $t>1$. However, the inequality $(7/4)^{n-1}<8$ holds only for $n=1,2,3,4$, but this gives an absurdity, since $n>\ell+1\ge 3+1=4$. Therefore $e^{\Lambda}\ne 1$.

Now, the conditions to apply Lemma 3 are fulfilled and hence

$$|e^{\Lambda}-1|>\exp(-1.5\cdot 10^{11}\ell^8(1+2\log\ell)\log(4\ell+4)(1+\log(m-1)))$$

Since, $1+2\log\ell\le 3\log\ell$ and $4\ell+4<\ell^{2.6}$ (for $\ell\ge 3$), we have that

$$|e^{\Lambda}-1|>\exp(-2.4\cdot 10^{12}\ell^8\log^2\ell\log(m-1)) \tag{11}$$

By combining (10) and (11), we get

$$\frac{m-1}{\log(m-1)}<6.1\cdot 10^{12}\ell^8\log^2\ell,$$

where we used that $\log\alpha>0.4$. Since the function $x/\log x$ is increasing for $x>e$, it is a simple matter to prove that

$$\frac{x}{\log x}<A\ \text{ implies that }\ x<2A\log A. \tag{12}$$

A proof for that can be found in [2, p. 74].

Thus, by using (12) for $x:=m-1$ and $A:=6.1\cdot 10^{12}\ell^8\log^2\ell$, we have that

$$m-1<2(6.1\cdot 10^{12}\ell^8\log^2\ell)\log(6.1\cdot 10^{12}\ell^8\log^2\ell).$$

Now, the inequality $30+2\log\log\ell<28\log\ell$, for $\ell\ge 3$, yields

$$\log(6.1\cdot 10^{12}\ell^8\log^2\ell)<30+8\log\ell+2\log\log\ell<36\log\ell.$$

Therefore

$$m<4.4\cdot 10^{14}\ell^8\log^3\ell \tag{13}$$

The proof is then complete. □

3. Upper bound for ℓ in terms of k

Lemma 4. *If* (m,n,ℓ,k) *is a solution in positive integers of equation* (2), *with* $\ell > k > 1, n > \ell+1$ *and* $m > k+1$, *then*

$$\ell < 1.8 \cdot 10^{16} k^3 \log^3 k. \tag{14}$$

Proof. If $\ell \leq 239$, then the inequalities (13) yields $m < 8 \cdot 10^{35}$. In particular, $\max\{m,n,\ell,k\} < 10^{36} < 2^{2000}$. So, by Lemma 1, the only solutions of equation (2) with the conditions in the statement of Theorem 1 are $(m,n,\ell,k) = (7,6,3,2)$ and $(12,11,7,3)$.

Thus, we may assume that $\ell > 239$. Therefore

$$n < 4.4 \cdot 10^{14} \ell^8 \log^3 \ell < 2^{\ell/2} \tag{15}$$

where we used (13) and the fact that $n < m$. By using a key argument due to Bravo and Luca [2, p. 72-73], we get

$$|2^{n-2} - g(\alpha,k)\alpha^{m-1}| < \frac{5 \cdot 2^{n-2}}{2^{\ell/2}} \tag{16}$$

or equivalently,

$$|1 - g(\alpha,k)\alpha^{m-1} 2^{-(n-2)}| < \frac{5}{2^{\ell/2}}. \tag{17}$$

For applying Lemma 3, it remains us to prove that the left-hand side of (17) is nonzero, or equivalently, $2^{n-2} \neq g(\alpha,k)\alpha^{m-1}$. To obtain a contradiction, we suppose the contrary, i.e., $2^{n-2} = g(\alpha,k)\alpha^{m-1}$. By conjugating the previous relation in the splitting field of $\psi_k(x)$, we obtain $2^{n-2} = g(\alpha_i,k)\alpha_i^{m-1}$, for $i = 1,\ldots,k$. However, when $i > 1$, $|\alpha_i| < 1$ and $|g(\alpha_i,k)| < 2$. But this leads to the following absurdity

$$2^{n-2} = |g(\alpha_i,k)||\alpha_i|^{m-1} < 2,$$

since $n > 4$. Therefore $g(\alpha,k)\alpha^{m-1}2^{-(n-2)} \neq 1$ and then we are in position to apply Lemma 3. For that, take $t := 3$,

$$\gamma_1 := g(\alpha,k),\ \gamma_2 := \alpha,\ \gamma_3 := 2$$

and

$$b_1 := 1,\ b_2 := m-1,\ b_3 := -(n-2).$$

By some calculations made in Section 2, we see that $A_1 := k\log(4k+4)$, $A_2 = A_3 := 0.7$ are suitable choices. Moreover $D = k$ and $B = m-1$. Thus

$$|1-g(\alpha,k)\alpha^{m-1}2^{-(n-2)}| > \exp(-C_1k^3(1+\log k)(1+\log(m-1))\log(4k+4)), \tag{18}$$

where we can take $C_1 = 0.75\cdot 10^{11}$. Combining (17) and (18) together with a straightforward calculation, we get

$$\ell < 4.7\cdot 10^{12}k^3\log^2 k\log m \tag{19}$$

On the other hand, $m < 4.4\cdot 10^{14}\ell^8\log^3\ell$ (by (13)) and so

$$\log m < \log(4.4\cdot 10^{14}\ell^8\log^3\ell) < 45\log\ell. \tag{20}$$

Turning back to inequality (19), we obtain

$$\frac{\ell}{\log\ell} < 2.2\cdot 10^{14}k^3\log^2 k$$

which implies (by (12)) that

$$\ell < 2(2.2\cdot 10^{14}k^3\log^2 k)\log(2.2\cdot 10^{14}k^3\log^2 k).$$

Since $\log(2.2\cdot 10^{14}k^3\log^2 k) < 39\log k$, we finally get the desired inequality

$$\ell < 1.8\cdot 10^{16}k^3\log^3 k.$$

□

4. The proof of Theorem 1

If $k \le 1655$, then $\ell < 4\cdot 10^{28}$ (by (14)). Thus, by (13), one has that $n < m < 2\cdot 10^{248}$. In particular, $\max\{m,n,\ell,k\} < 2\cdot 10^{248} < 2^{2000}$. So, Lemma 1 gives the known solutions.

Therefore, we may suppose that $k > 1655$. The inequality $\ell < 1.8\cdot 10^{16}k^3\log^3 k$ together with (13) yield

$$\begin{aligned} m &< 4.4\cdot 10^{14}(1.8\cdot 10^{16}k^3\log^3 k)^8\log^3(1.8\cdot 10^{16}k^3\log^3 k)\\ &< 3\cdot 10^{148}k^{24}\log^{27}k < 2^{k/2}, \end{aligned}$$

where the last inequality holds only because $k > 1655$. Now, we use again the key argument of Bravo and Luca to conclude that

$$|2^{m-2} - g(\phi, \ell)\phi^{n-1}| < \frac{5 \cdot 2^{m-2}}{2^{k/2}}. \qquad (21)$$

Combining (16), (21) and (8), we get

$$\begin{aligned} |2^{n-2} - 2^{m-2}| &\leq |2^{n-2} - g(\alpha, k)\alpha^{n-1}| + |g(\alpha, k)\alpha^{n-1} - g(\phi, \ell)\phi^{n-1}| \\ &\quad + |2^{m-2} - g(\phi, \ell)\phi^{n-1}| \\ &< \frac{5 \cdot 2^{n-2}}{2^{\ell/2}} + 1 + \frac{5 \cdot 2^{m-2}}{2^{k/2}} < \frac{11 \cdot 2^{m-2}}{2^{k/2}}, \end{aligned}$$

since $n < m$, $k < \ell$ and $m > k + 1$. Therefore

$$|2^{n-m} - 1| < \frac{11}{2^{k/2}}. \qquad (22)$$

Since $n \leq m - 1$, then

$$\frac{1}{2} \leq 1 - 2^{n-m} = |2^{n-m} - 1| < \frac{11}{2^{k/2}}.$$

Thus $2^{k/2} < 22$ leading to an absurdity, since $k > 1655$.

In conclusion, the only solutions of equation (2) with $\ell > k > 1, n > \ell + 1$ and $m > k + 1$ are those listed in (3). Thus, the proof of Theorem 1 is complete. □

5. The program

In this section, for the sake of completeness, we present the Mathematica program (which was kindly sent to us by Noe [19]) used to confirm Lemma 1:

```
nn = 2000;
f = 2^Range[nn] - 1;
f[[1]] = Infinity;
cnt = 0;
seq = Table[Join[2^Range[i - 1], {2^i - 1}], {i, nn}];
done = False;
While[! done, fMin = Min[f];
```

```
pMin = Flatten[Position[f, fMin]];
If[Length[pMin] > 1, Print[{fMin, pMin}]];
Do[k = pMin[[i]];
 s = Plus @@ seq[[k]];
 seq[[k]] = RotateLeft[seq[[k]]];
 seq[[k, k]] = s;
 f[[k]] = s, {i, Length[pMin]}];
cnt++;
done = (fMin > 2^nn)]; cnt
```

The calculations took roughly 54 hours on 1.80 GHz AMD Triple-Core PC.

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