Generalized Weingarten Surfaces of the Radial Support Type

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Abstract. I intend to do a poster submission about surfaces that can be parametrized as envelopes of a sphere congruence whose other envelope is contained in a unit sphere. Such surfaces has a local parametrization given by

\[ X(u) = Y(u) - 2 \left( \frac{h(u) + c}{S(u)} \right) \eta(u) \quad u \in U, \]  

(1)

where \( h : U \subset \mathbb{R}^n \to \mathbb{R} \) is a real differentiable function associated with the parametrization \( Y : U \subset \mathbb{R}^n \to \mathbb{S}^n \) of \( \mathbb{S}^n \) and

\[ \eta = \nabla_L h + h Y, \quad S = \langle \eta, \eta \rangle = |\nabla_L h|^2 + h^2, \]

with \( L_{ij} = \langle Y_i, Y_j \rangle \).

In my work it is exhibited a sufficient condition to exist such sphere congruence. Moreover its radius function is given explicitly and it is proved to be a geometric invariant of the surface.

The characterization of surfaces that are associated to \( \mathbb{S}^n \) by a sphere congruence is used to study a class of generalized Weingarten surfaces, named generalized Weingarten surfaces of the radial support type - RSGW- surfaces in short - which satisfy a differentiable relation between the mean and Gaussian curvatures, the support function and the radius function from the sphere congruence.

Under certain condition, a surface locally parametrized as in (1) is a RSGW-surface if, and only if, the function \( h \) is harmonic. In this case, the vector function \( \eta \) is a local parametrization for an Appell surface. Furthermore, there is a Weierstrass type representation depending on two holomorphic functions for the RSGW-surfaces and, consequently, for the Appell surfaces locally parametrized by \( \eta \).

References


