Associative Anti-Unification

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Summary

About the development of work

This presentation is based in my masters final work, Syntactc, Commutative and Associative Anti-Unification, that was presented in the final of the last year and was supervised by Daniele Nantes.

Universidade de Brasília

Syntactic, Commutative and Associative Anti-Unification

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Departamento de Matemática
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Master in Mathematics

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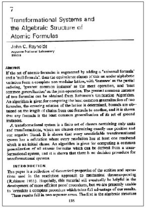
This talk

- We will present the Anti-Unification Problem modulo empty ($\emptyset$) and associative (A) theories;
- We will present algorithms $\texttt{AUnif}_E$ based on simplification rules for each of these cases:
  - pointing out the different results obtained for each equational theory,
  - give examples;
- Analyse the termination, confluence and correctness properties of the anti-unification algorithms,
  - with an especial attention on the prove of completeness of $\texttt{AUnif}_A$, that is different from the original approach in [AEEM14].

History



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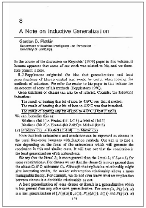
Transformational Systems and the Algebraic Structure of Atomic Formulas

John C. Reynolds



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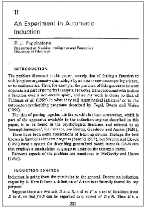
A Note on Inductive Generalization

Gordon D. Plotkin



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An Experiment in Automatic Induction

R. J. Popplestone

Unification, Weak Unification, Upper Bound, Lower Bound, and Generalization Problems

Franz Baader

Information and Computation

A modular order-sorted equational generalization algorithm

First notions:

- Popplestone [Pop70],
- Plotkin [Plo70],
- and Reynolds [Rey70],
- Machine Intelligence Journal.

Important results:

- Existence of the solutions of the Syntactic, Commutative and Associative Anti-Unification Problems [Baa91],
- Development of methods to solve these problems [AEEM14].

Syntax

Before define the Anti-Unification Problem we need to define some basic concepts

Finite Signature: $\sigma = \Sigma_\emptyset \cup \Sigma_A \cup \Sigma_C$.

- $\Sigma_\emptyset = \{\underbrace{a : 0, b : 0, c : 0, d : 0}_{\text{constants}}, f : n, \ldots\}$ without an equational theory.
- $\Sigma_A = \{h : 2\}$ with the associative function symbol

$$A = \{h(x, h(y, z)) \approx h(h(x, y), z)\}.$$

Generalizer

Definition (Generalizer)

Given two terms $s, t \in T(\mathcal{X}, \Sigma_\emptyset)$. A **generalizer** of s and t is a term $r \in T(\mathcal{X}, \Sigma)$ for which there exists a pair of substitutions $\overline{\theta} = (\theta_1, \theta_2)$ such that $r\theta_1 = s$ and $r\theta_2 = t$.

$$\mathtt{gen}(s, t).$$

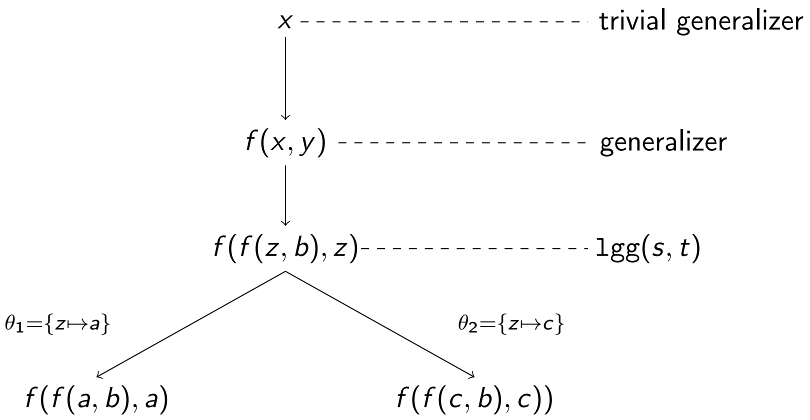
Definition (Least General Generalization)

Given a signature Σ and terms s and $t \in T(\mathcal{X}, \Sigma)$. We define the **the least general generalization** of s and t as the greatest lower bound generalizer of s and t. In other words:

$$\mathtt{lgg}(s, t) = \{r \in \mathtt{gen}(s, t) \mid r' \leq r, \forall r' \in \mathtt{gen}(s, t)\}$$

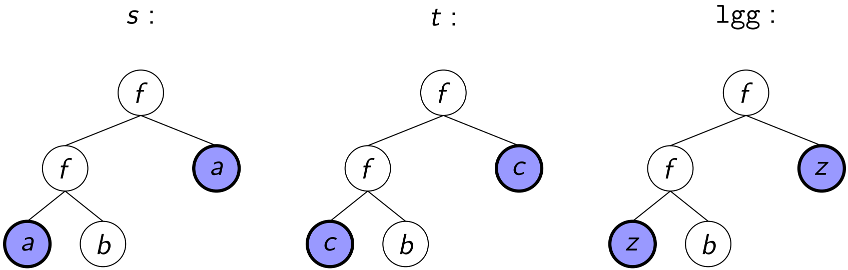
Example - Generalizers

$s = f(f(a, b), a)$ and $t = f(f(c, b), c)$



Comparing term structures

The least general generalizer of s and t is the generalizer which maintains more the structure of s and t as possible.



Syntactic Anti-Unification Problem (AUP)

Definition ($\mathcal{A}\langle s, t\rangle$)

- **Given:** terms s and $t \in T(\mathcal{X}, \Sigma_\emptyset)$,
- **Find:** The least general generalizer of s and t.

Example

Let $s = f(f(a, b), a)$ and $t = f(f(c, b), c)$. Then, $r = f(f(z, b), z)$ it is an solution for the AUP $\mathcal{A}\langle s, t\rangle$.

A rule based anti-unification algorithm

Input: $\mathcal{A}\langle s, t\rangle$

$\texttt{AUnif}_{\emptyset} - \mathit{rules}$

$$\langle \{x : s \triangleq t\} \mid \emptyset \mid id\rangle \Longrightarrow \dots \Longrightarrow \langle \emptyset \mid S \mid \sigma\rangle$$

initial state

final state

Output: $x\sigma \in \mathtt{lgg}(s, t)$

$\texttt{AUnif}_{\emptyset}$ rules

(Dec) **: Decompose:**

$$\langle P \cup \{x : f(\overline{s_n}) \triangleq f(\overline{t_n})\} \mid S \mid \sigma\rangle \Longrightarrow \langle P \cup \left\{\begin{matrix} x_1 : s_1 \triangleq t_1, \\ \vdots \\ x_n : s_n \triangleq t_n \end{matrix}\right\} \mid S \mid \sigma\{x \mapsto f(\overline{x_n})\}\rangle$$

where $x_1, \ldots, x_n$ are fresh variables.

(Sol)**: Solve:** If $\texttt{root}(s) \neq \texttt{root}(t)$ and there is no constraint $\{y : s \triangleq t\} \in S$

$$\langle P \cup \{x : s \triangleq t\} \mid S \mid \sigma\rangle \Longrightarrow \langle P \mid S \cup \{x : s \triangleq t\} \mid \sigma\rangle$$

(Rec)**: Recover:** If $\texttt{root}(s) \neq \texttt{root}(t)$

$$\langle P \cup \{x : s \triangleq t\} \mid S \cup \{y : s \triangleq t\} \mid \sigma\rangle \Longrightarrow \langle P \mid S \cup \{y : s \triangleq t\} \mid \sigma\{x \mapsto y\}\rangle$$

Example

Consider the AUP $\mathcal{A}\langle s, t\rangle$ for $s = f(f(a, b), a)$ and $t = f(f(c, b), c)$.

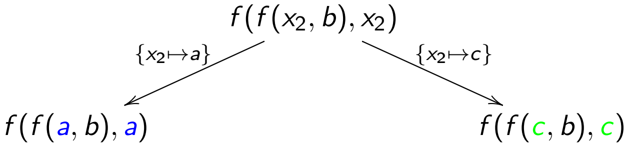
$$\langle \{x : f(f(a, b), a) \triangleq f(f(c, b), c)\} \mid \emptyset \mid id \rangle$$
$$\Downarrow (Dec)$$
$$\langle \{x_1 : f(a, b) \triangleq f(c, b), x_2 : a \triangleq c\} \mid \emptyset \mid \underbrace{\{x \mapsto f(x_1, x_2)\}}_{\sigma_1} \rangle$$
$$\Downarrow (Sol)$$
$$\langle \{x_1 : f(a, b) \triangleq f(c, b)\} \mid \{x_2 : a \triangleq c\} \mid \sigma_1 \rangle$$
$$\Downarrow (Dec)$$
$$\langle \{x_3 : a \triangleq c, x_4 : b \triangleq b\} \mid \{x_2 : a \triangleq c\} \mid \underbrace{\sigma_1\{x_2 \mapsto f(x_3, x_4)\}}_{\sigma_2} \rangle$$
$$\Downarrow (Rec)$$
$$\langle \{x_4 : b \triangleq b\} \mid \{x_2 : a \triangleq c\} \mid \underbrace{\sigma_1\{x_3 \mapsto x_2\}}_{\sigma_3} \rangle$$
$$\Downarrow (Dec)$$
$$\langle \emptyset \mid \{x_2 : a \triangleq c\} \mid \underbrace{\sigma_1\{x_4 \mapsto b\}}_{\sigma_4} \rangle$$

Example - Continuation

Consider the AUP $\mathcal{A}\langle s, t\rangle$ for $s = f(f(a,b),a)$ and $t = f(f(c,b),c)$.

$$\langle \{x : f(f(a,b),a) \triangleq f(f(c,b),c)\} \mid \emptyset \mid id\rangle \overset{*}{\Longrightarrow}_{\mathtt{AUnif}_\emptyset} \langle \emptyset \mid \{x_2 : a \triangleq c\} \mid \underbrace{\sigma_1\{x_4 \mapsto b\}}_{\sigma_4}\rangle$$

Then, $\mathtt{AUnif}_\emptyset(s,t)$ gives $x\sigma_4 = f(f(x_2,b),x_2)$.



Properties of $\texttt{AUnif}_\emptyset$

Let $\mathcal{A}\langle s, t\rangle$ be an AUP,

- Termination $\texttt{AUnif}_\emptyset$ terminates
- Confluence $\texttt{AUnif}_\emptyset$ has a unique normal form except for variable renaming,
- Correctness $r \in \texttt{lgg}(s, t)$ iff there exists a derivation

$$\langle \{x : s \triangleq t\} \mid \emptyset \mid id\rangle \overset{*}{\Longrightarrow}_{\texttt{AUnif}_\emptyset} \langle \emptyset \mid S \mid \sigma\rangle$$

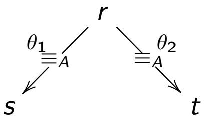
  such that $x\sigma \equiv r$.

Therefore $\mathcal{A}\langle s, t\rangle$ always will have a solution that is unique, except for variable renaming.

The Associative Anti-Unification Problem

Definition (Associative Anti-Unification Problem - AUP_A)

- **Given:** Two terms s and $t \in T(\mathcal{X}, \Sigma_{\emptyset \cup A})$,
- **Find:** The set $\mathtt{lgg}_A(s, t)$.



The AUP_A for s and t is denoted by $\mathcal{A}_A\langle s, t\rangle$.

Flattening

Given h the associative function symbol with $n \leq 2$ arguments, flattened terms are canonical forms w.r.t. the set of rules given by the following rule schema

$$h(x_1, \ldots, h(t_1, \ldots, t_n), \ldots, x_n) \longrightarrow h(x_1, \ldots, t_1, \ldots, t_n, \ldots, x_n)$$

$\mathtt{AUnif}_A$: simplification rules

(Dec): Decompose ($f \in \Sigma_\emptyset$ and $x \in \mathcal{X}$)

(Sol): Solve

(Rec): Recover

(A-Dec): Associative Decompose

- (A-Left)
- (A-Right)

Associative for left and right sides

(A-Left) Associative-Left Decompose

$$\langle P \cup \{x : h(s_1, \dots, s_n) \triangleq h(t_1, \dots, t_m)\} \mid S \mid \sigma\rangle$$

$$\Longrightarrow \langle P \cup \left\{ \begin{array}{l} x_1 : h(s_1, \dots, s_k) \triangleq t_1 \\ x_2 : h(s_{k+1}, \dots, s_n) \triangleq h(t_2, \dots, t_m) \end{array} \right\} \mid S \mid \sigma\{x \mapsto h(x_1, x_2)\}\rangle$$

with $k \leq n - 1$.

(A-Right) Associative-Right Decompose

$$\langle P \cup \{x : h(s_1, \dots, s_n) \triangleq h(t_1, \dots, t_m)\} \mid S \mid \sigma\rangle$$

$$\Longrightarrow \langle P \cup \left\{ \begin{array}{l} x_1 : s_1 \triangleq h(t_1, \dots, t_k) \\ x_2 : h(s_2, \dots, s_n) \triangleq h(t_{k+1}, \dots, t_m) \end{array} \right\} \mid S \mid \sigma\{x \mapsto h(x_1, x_2)\}\rangle$$

with and $1 < k \leq m - 1$.

Example: AUP_A.

Consider $\mathcal{A}_A\langle s, t\rangle$ an AUP_A with $s = h(h(a,a), h(b,c))$ and $t = h(h(b,b), c)$.

term	$s =$	$h(h(a,a), h(b,c))$	$s =_A$	$h(a, h(a, h(b,c)))$
term	$t =$	$h(h(b,b), c)$	$t =$	$h(h(b,b), c)$
generalizer	$r_1 =$	$h(h(x,x), y)$	$r_2 =$	$h(x, y)$
term	$s =$	$h(h(a,a), h(b,c))$	$s =_A$	$h(a, h(a, h(b,c)))$
term	$t =_A$	$h(b, h(b,c))$	$t =_A$	$h(b, h(b,c))$
generalizer	$r_3 =$	$h(x, h(b,c))$	$r_4 =$	$h(x, h(x,y))$

Notice that

- $r_2 <_A r_1, r_3$ and r_4;
- $r_1 \equiv_A r_4$.

Example

To apply the rules of $\mathtt{AUnif}_A$ to solve this problem we first put s and t in they flattened form.

$$h(h(a,a),h(b,c)) \overset{\text{flattening}}{\dashrightarrow} h(a,a,b,c)$$

$$h(h(b,b),c) \overset{\text{flattening}}{\dashrightarrow} h(b,b,c)$$

Example

$$\langle \{x : h(a,a,b,c) \triangleq h(b,b,c)\} \mid \emptyset \mid id \rangle$$

$(A\text{-}Left)_{k=1}$:

$$C_1 = \langle \left\{ \begin{array}{l} x_1 : a \triangleq b \\ x_2 : h(a,b,c) \triangleq h(b,c) \end{array} \right\} \mid \emptyset \mid \underbrace{\{x \mapsto h(x_1,x_2)\}}_{\sigma_1} \rangle$$

$\Downarrow$ (Sol), $(A\text{-}Left)$

$$\langle \left\{ \begin{array}{l} x_3 : a \triangleq b, \\ x_4 : h(b,c) \triangleq c \end{array} \right\} \mid \{x_1 : a \triangleq b\} \mid \underbrace{\sigma_1\{x_2 \mapsto h(x_3,x_4)\}}_{\sigma_2} \rangle$$

$\Downarrow$ (Sol), (Sol)

$$\langle \emptyset \mid \left\{ \begin{array}{l} x_1 : a \triangleq b \\ x_4 : h(b,c) \triangleq c \end{array} \right\} \mid \underbrace{\sigma_2\{x_3 \mapsto x_1\}}_{\sigma_3} \rangle$$

$(A\text{-}Left)_{k=2}$:

$$\langle \left\{ \begin{array}{l} x_1 : h(a,a) \triangleq b \\ x_2 : h(b,c) \triangleq h(b,c) \end{array} \right\} \mid \emptyset \mid \sigma_1 \rangle$$

$\Downarrow$ $(A\text{-}Left)$

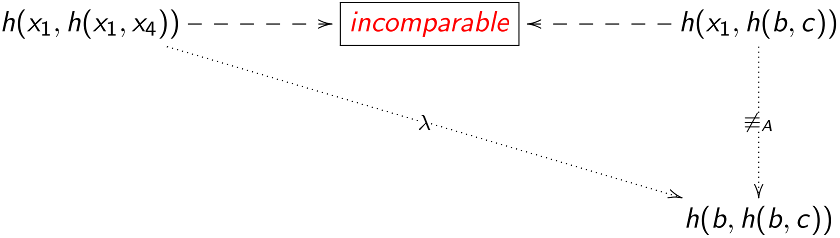
$$\langle \left\{ \begin{array}{l} x_1 : h(a,b) \triangleq b, \\ x_3 : b \triangleq b, \\ x_4 : c \triangleq c \end{array} \right\} \mid \emptyset \mid \sigma_2 \rangle$$

$\Downarrow$ (Sol), (Dec), (Dec)

$$\langle \emptyset \mid \{x_1 : h(a,a) \triangleq b\} \mid \underbrace{\sigma_2\{x_3 \mapsto b, x_4 \mapsto c\}}_{\sigma_4} \rangle$$

Therefore, $\mathtt{AUnif}_A$ gives:

- $x\sigma_3 = h(x_1, h(x_1, x_4)) \equiv_A r_1$,
- $x\sigma_4 = h(x_1, h(b, c)) \equiv_A r_3$,



Therefore, $\mathtt{AUnif}_A$ is not confluent.

Properties of $\mathtt{AUnif}_A$

Let $\mathcal{A}_A\langle s, t\rangle$ be an AUP_A.

- Terminates $\mathtt{AUnif}_A$ terminates.
- Sound If $\langle\{x : s \triangleq t\} \mid \emptyset \mid id\rangle \overset{*}{\Longrightarrow}_{\mathtt{AUnif}_C} \langle\emptyset \mid S \mid \sigma\rangle$ then $x\sigma \in \mathrm{gen}_A(s, t)$.
- Complete If $r \in \mathtt{lgg}_A(s, t)$, then there exists a derivation

$$\langle\{x : s \triangleq t\} \mid \emptyset \mid id\rangle \overset{*}{\Longrightarrow}_{\mathtt{AUnif}_A} \langle\emptyset \mid S \mid \sigma\rangle$$

such that $x\sigma \equiv_A r$.

There was a problem in the original proof of Completeness of $\mathtt{AUnif}_A$ in [AEEM14].

Notions

Before explain this problem we need to establish some notions.

Definition (Associative pair of positions)

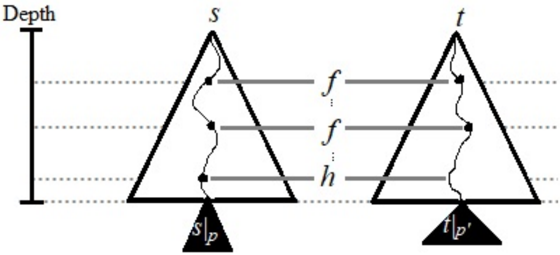


Figure: Caption

Definition (Associative Pair of Subterms)

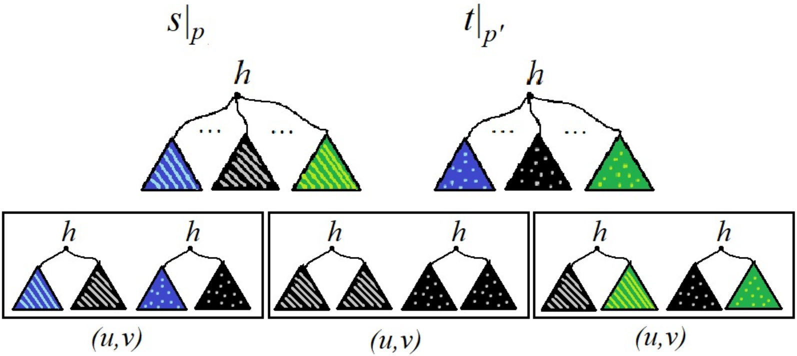
Let $s, t \in T(\mathcal{X}, \Sigma_{\emptyset \cup A})$ be terms in flattened form. The pair of terms (u, v) is called an **associative pair of subterms** of s and t iff

1. **(Regular Subterms)** For each pair of positions $p \in \mathrm{pos}(s)$ and $p' \in \mathrm{pos}(t)$ such that $s|_p = u, t|_{p'} = v$ and (p, p') is an associative pair of positions of s and t.

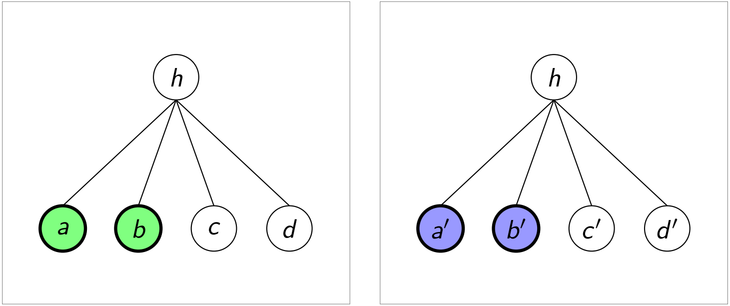
   Or:

Definition (Associative pair of subterms)

2 **(Associative pair of subterms)** There are positions an associative pair of positions (p, p') such that



Example



Given terms $s = h(a, b, c, d)$ and $t = h(a', b', c', d')$, it follows $(h(a, b), h(a', b'))$ is an associative pair of s and t.

Proof of Completeness given by [AEEM14]

Now we can explain the problem in the proof.

Lemma (c.f. Lemma 19 in [AEEM14])

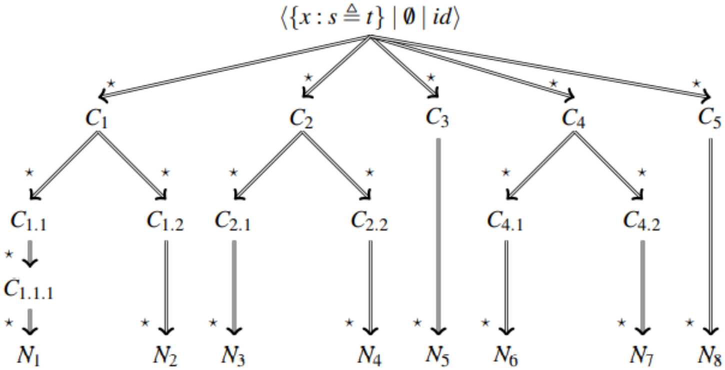
Given flattened terms t and t' such that every symbol in t and t' is either free or associative, and a fresh variable x, then there is a sequence

$$\langle\{y : t \triangleq t'\} \mid \emptyset \mid id\rangle \overset{*}{\Longrightarrow}_{\mathtt{AUnif}_A} \langle P \cup \{y : u \triangleq v\} \mid S \mid \sigma\rangle$$

such that there is no variable z such that $\{z : u \triangleq v\} \in S$ if and only if (u, v) is an associative pair of subterms of t and t'.

Counter example: Simplification tree

Let $\mathcal{A}_A\langle s, t\rangle$ be an AUP$_A$, where $s = h(a, b, c, d)$ and $t = h(a', b', c', d')$, as applying the simplification rules of $\texttt{AUnif}_A$ to solve this problem we obtain the following simplification tree:



Counter example: Description of configurations

$$C_1 = \langle\{x_2 : h(b,c,d) \triangleq h(b',c',d')\} \mid \{x_1 : a \triangleq a'\} \mid \{x \mapsto h(x_1,x_2)\}\rangle,$$

$$C_2 = \langle\{x_2 : h(c,d) \triangleq h(b',c',d')\} \mid \{x_1 : h(a,b) \triangleq a'\} \mid \{x \mapsto h(x_1,x_2)\}\rangle,$$

$$C_3 = \langle\{x_1 : h(a,b,c) \triangleq a', x_2 : d \triangleq h(b',c',d')\} \mid \emptyset \mid \{x \mapsto h(x_1,x_2)\}\rangle,$$

$$C_4 = \langle\{x_2 : h(b,c,d) \triangleq h(c',d')\} \mid \{x_1 : a \triangleq h(a',b')\} \mid \{x \mapsto h(x_1,x_2)\}\rangle,$$

$$C_5 = \langle\{x_1 : a \triangleq h(a',b',c'), x_2 : h(b,c,d) \triangleq d'\} \mid \emptyset \mid \{x \mapsto h(x_1,x_2)\}\rangle,$$

$$C_{1.1} = \langle\{x_3 : b \triangleq b', x_4 : h(c,d) \triangleq h(c',d')\} \mid \{x_1 : a \triangleq a'\}\{x \mapsto h(x_1,h(x_3,x_4))\}\rangle,$$

$$C_{1.2} = \langle\{x_3 : h(b,c) \triangleq b', x_4 : d \triangleq h(c',d')\} \mid \{x_1 : a \triangleq a'\} \mid \{x \mapsto h(x_1,h(x_3,x_4))\rangle,$$

$$C_{2.1} = \langle\{x_3 : c \triangleq b', x_4 : d \triangleq h(c,d')\} \mid \{x_1 : h(a,b) \triangleq a'\} \mid \{x \mapsto h(x_1,h(x_3,x_4))\rangle,$$

$$C_{2.2} = \langle\{x_3 : c \triangleq h(b',c'), x_4 : d \triangleq d' \mid \{x_1 : h(a,b) \triangleq a'\} \mid \{x \mapsto h(x_1,h(x_3,x_4))\rangle,$$

$$C_{4.1} = \langle\{x_3 : b \triangleq c, x_4 : h(c,d) \triangleq d'\} \mid \{x_1 : a \triangleq h(a,b')\} \mid \{x \mapsto h(x_1,h(x_3,x_4))\rangle,$$

$$C_{4.2} = \langle\{x_3 : h(b,c) \triangleq c', x_4 : d \triangleq d'\} \mid \{x_1 : a \triangleq h(a,b')\} \mid \{x \mapsto h(x_1,h(x_3,x_4))\rangle;$$

$$C_{1.1.1} = \langle\{x_5 : c \triangleq c', x_6 : d \triangleq d'\} \mid \{x_1 : a \triangleq a', x_3 : b \triangleq b'\} \mid \{x \mapsto h(x_1,h(x_3,h(x_5,x_6)))\}\rangle.$$

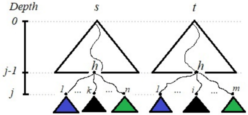
Counter example: conclusion

- There is no configuration $\langle P \cup \{y : h(a, b) \triangleq h(a', b')\} \mid S \mid \sigma\rangle$ in the simplification tree of $\texttt{AUnif}_A(s, t)$;
- Lemma 19 in [AEEM14] does not hold!

This lemma is used to prove the completeness of $\texttt{AUnif}_A$ in [AEEM14].
In order to show that this property still holds, we replace the Lemma 19 in [AEEM14] for tree new lemmas (Lemma 4.2, 4.3 and 4.4) that will be stated in the following frames.

Lemma (4.2)

Let $\mathcal{A}_A\langle s, t\rangle$ an AUP_A. If (p, p') is an associative pair of positions of s and t, then there exists a derivation of the form
$\langle\{x : s \triangleq t\} \mid \emptyset \mid id\rangle \overset{*}{\Longrightarrow}_{\mathtt{AUnif}_A} \langle\{y : u \triangleq v\} \mid S \mid \sigma\rangle$ *with* $(s|_p, t|_{p'}) = (u, v)$.



- Relates the arguments of the flattened terms with the configurations of $\mathtt{AUnif}_A$.

Lemma (4.3)

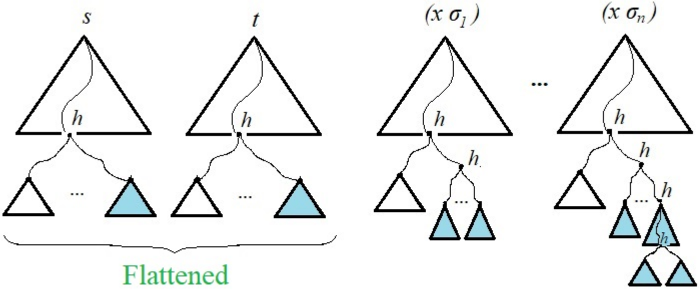
Let $\mathcal{A}_A\langle s, t\rangle$ *be an* AUP_A *and* (p, p') *an associative pair of positions of* s *and* t*, such that*

$$s|_p = h(s_1, \dots, s_k, u_1, \dots, u_n, s_{k+1} \dots, s_q),$$

$$t|_{p'} = h(t_1, \dots, s_{k'}, v_1, \dots, v_m, t_{k'+1}, \dots, s_{q'}).$$

If $(u, v) = (h(\overline{u_n}), h(\overline{v_m}))$ *is an associative pair of subterms, then there exists derivations such that*

1. $\langle \{x : s \triangleq t\} \mid \emptyset \mid id\rangle \overset{*}{\Longrightarrow}_{\texttt{AUnif}_A} \langle P \cup \{y : h(u_1, \dots, u_i) \triangleq v_1\} \mid S \mid \sigma\rangle$ *with* $1 \leq i \leq n - 1$*, and*
2. $\langle \{x : s \triangleq t\} \mid \emptyset \mid id\rangle \overset{*}{\Longrightarrow}_{\texttt{AUnif}_A} \langle P \cup \{y : u_1 \triangleq h(v_1, \dots, v_j)\} \mid S \mid \sigma\rangle$ *with* $1 < j \leq m - 1$.



- Relates the associative pairs of subterms with the configurations of the derivations of $\mathtt{AUnif}_A(s, t)$.

Lemma (4.4)

Let $\mathcal{A}_A\langle s,t\rangle$ *be an* AUP_A*. If there exists a sequence of the form*

$$\langle \{x : s \triangleq t\} \mid \emptyset \mid id\rangle \overset{*}{\Longrightarrow}_{\mathtt{AUnif}_A} \langle P \cup \{y : u \triangleq v\} \mid S \mid \sigma\rangle$$

then (u,v) *is an associative pair of subterms of* s *and* t*.*

Conclusion

We have verified that:

problem	algorithm	terminating	confluent	sound	complete
AUP	$\texttt{AUnif}_{\emptyset}$	✓	✓	✓	✓
AUP_A	$\texttt{AUnif}_A$	✓	×	✓	✓

- AUP always have a unique solution (except for variable renaming)
- AUP_A always have a finite and minimal set of solutions (but $\texttt{AUnif}_A$ do not gives minimal solutions),

Future work

- To obtain measure that gives a maximal bound of the number of normal forms obtained by $\Longrightarrow_{\texttt{AUnif}_C}$ and $\Longrightarrow_{\texttt{AUnif}_A}$.
- To extend the study of AUP_A for high order context of Nominal Framework, extending the work by Baumgartner et. al. in [BKLV15].

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