Departamento de Matematica

Lista 8 - Variável Complexa 1

1. Calcular $\int_\gamma f(z)\,dz$, onde

   a) $f(z) = \dfrac{1}{z - z_0}$ e $\gamma(t) = z_0 + re^{it}, 0 \leq t \leq 2\pi, r > 0.$

   b) $f(z) = \dfrac{1}{(z - z_0)^n}$ e $\gamma(t) = z_0 + re^{it}, 0 \leq t \leq 2\pi, r > 0, n \geq 2.$

   c) $f(z) = \dfrac{e^{iz}}{z^2}$ e $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi.$

   d) $f(z) = \dfrac{\cos z}{z^4}$ e $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi.$

   e) $f(z) = \dfrac{\log z}{z^n}$ e $\gamma(t) = 1 + \frac{1}{4}e^{it}, 0 \leq t \leq 2\pi.$

   f) $f(z) = \dfrac{\cosh z}{z^n}$ e $\gamma(t) = 2e^{it}, 0 \leq t \leq 2\pi, n \geq 1.$

   g) $f(z) = \dfrac{1}{z^2 + 1}$ e $\gamma(t) = 2e^{it}, 0 \leq t \leq 2\pi.$

2. Seja $\gamma(t) = re^{it}, 0 \leq t \leq \pi$ e considere a seguinte integral

$$I(r) = \int_\gamma \frac{e^{iz}}{z}\,dz.$$

   Calcule $\lim\limits_{r \to \infty} I(r)$.

3. Mostre que

$$\int_\gamma \frac{e^{kz}}{z}\,dz = 2\pi i,$$

   onde k é uma constante real e $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$. Use este resultado para mostrar que

$$\int_0^\pi e^{k\cos t} \cos(k\ \mathrm{sen}\, t)\,dt = \pi.$$