

ANALYSIS SEMINAR

Existence of Solutions to a Weakly Coupled Logistic System and their Asymptotic Behavior**Mayra Soares**

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Abstract. We study the weakly coupled elliptic system of logistic type in the form of

$$\begin{cases} -\Delta u = \lambda_1 u - u^3 + \beta uv^2 & \text{in } \Omega, \\ -\Delta v = \lambda_2 v - v^3 + \beta u^2 v & \text{in } \Omega, \\ u, v \in H_0^1(\Omega), \end{cases} \quad (1)$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with $N \geq 2$. We say the system is competitive if $\beta < 0$ and cooperative if $\beta > 0$, $\beta \in \mathbb{R}$, $\lambda_1(\Omega) < \lambda_1 \leq \lambda_2$. We prove the existence and multiplicity of solutions to the problem (1) in alternative variational frameworks depending on the information we have about the parameter β . We do not rely on bifurcation or degree theory, which have been used in the literature for logistic-type problems. Instead, the novelty is to obtain min-max type solutions by exploring the different geometry of the functional associated with the logistic problem. Furthermore, we analyze the asymptotic behavior of such solutions as $\beta \rightarrow 0$ or $\beta \rightarrow \pm\infty$.

References

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