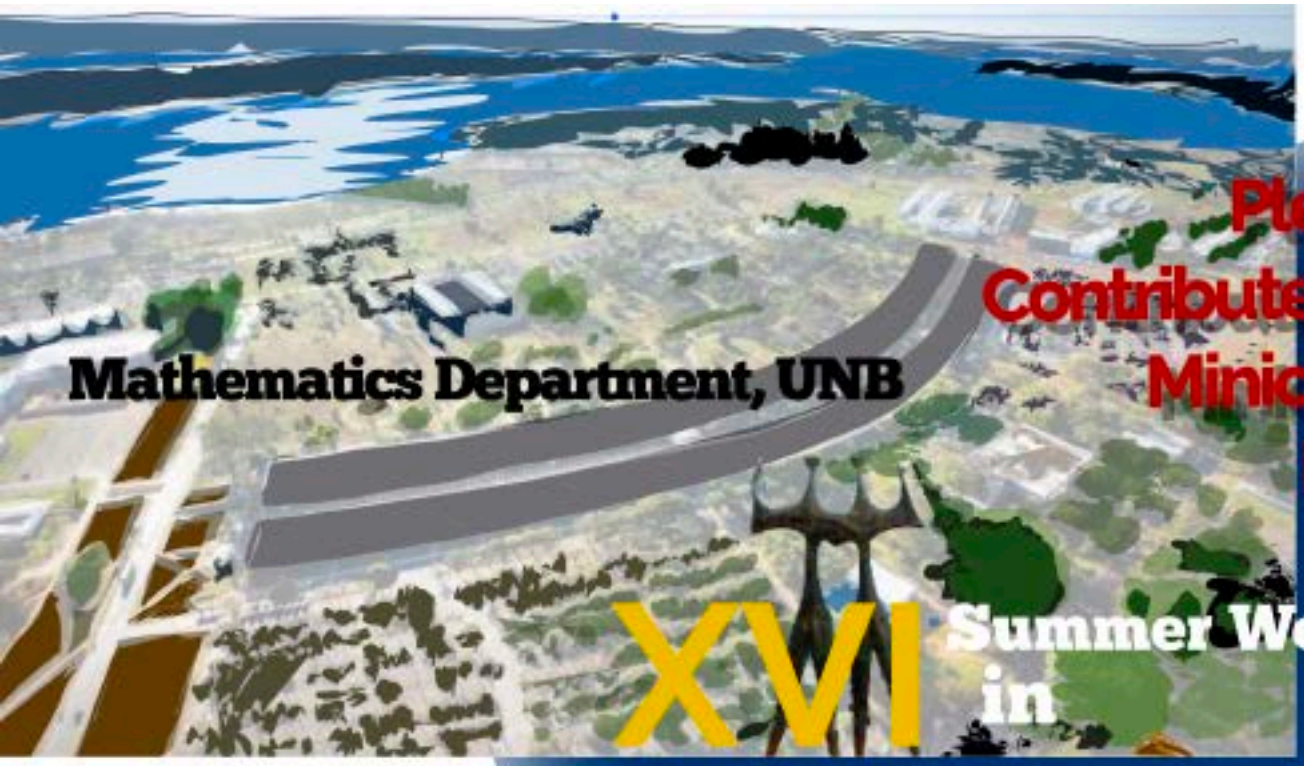

Plenaries,
Contributed Talks
Minicourses,
Posters

XVI Summer Workshop in MATHEMATICS

February 05th to 09th, 2024

Coordinators:

Profa. Sheila Chagas (UnB)
Prof. Tarcísio Silva (UnB)

Scientific Committee

Prof. Ashot Minasyan (University of Southampton-UK)
Prof. Pavel Shumyatsky (UnB)
Profa. Eloisa Demoti (Universidade de Padova-it)
Prof. Liliane Maia (UnB)
Prof. Ma To Fu (UnB)
Profa. Keti Tenenblat (UnB)
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Welcome to

XVI Summer Workshop in Mathematics

Summer Workshop in Mathematics

Book of Abstracts

Summer Workshop in Mathematics

Coordinator

Sheila Chagas (University of Brasília, Brazil)
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Scientific Committee

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Prof. Paulo H. P. da Costa - Probabilidade
Prof. Yuri Dumaresq Sobral - Mecânica
Prof. José Antônio O. de Freitas - Apoio Técnico

Opening Ceremony: Monday, January 05, from 08:30 to 9:00

Plenary Speakers

Ashot Minasyan (University of Southampton, United Kingdom)
Eloisa Detomi (Universidade de Padova, Italy)
Vanderson Lima (UFRGS, Brazil)

Mónica Clapp (UNAM, México)
Benjamín Callejas Bedregal (UFRN, Brazil)
David M. Cerna (Czech Academy of Sciences, Czechia)
Michael Anton Hoegele (Universidad de los Andes, Colombia)
Fabrício Simeoni de Sousa (USP, Brazil)
Alessandro Jacques Ribeiro (Universidade Federal do ABC, Brazil)

Special Sessions

Algebra

Analysis

Geometry

Logic and Computation

Mathematical Education

Mechanics

Number Theory

Probability

Posters

Social Activities

Event Photo
The official event photo will be taken on January 7th at 4pm

Event Dinner
The event dinner will take place at the Sal And Brasa , January 7, from 7pm.

Event address

University of Brasília
Campus Universitário Darcy Ribeiro, Brasília-DF, 70910-900

Contents

General Information

Schedule

PLENARY LECTURES, OPENING AND CLOSING Main Room: FT - Auditorium					
	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
8h30	Opening FT-Auditorium				
9hs		9h-10h Registration	9h-10 Registration	9h-10 Registration	9h-10 Registration
9h30 - 10h30	**Eloisa Detomi** Universidade de Padova, Italy Álgebra	**Vanderson Lima** UFRGS - Brazil Geometria	**Ashot Minasyan** University of Southampton, UK Álgebra	**David M. Cerna** Czech Academy of Sciences, Czechia Computação	**Benjamin Callejas Bedregal** UFRN - Brazil Computação
10h30 - 11h	**Coffee Break**				
11h - 12h	**Mónica Clapp** UNAM - México Análise	**Fabricio Simeoni de Sousa** USP - Brazil Mecânica	**Alessandro Jacques Ribeiro** Universidade Federal do ABC - Brazil Educação	**Michael Anton Hoegele** Universidad de los Andes - Colombia Probability	**10h Closing** **FT - Auditorium**
12h - 14h	**Lunch Time**				
14h - 16h	**Thematic Sessions**	**Thematic Sessions**	**16h - Event Photo**	**Thematic Sessions**	**Thematic Sessions**
16h - 16h30	**Coffee Break / Poster Session**				
16h30 - 18h30	**Thematic Sessions**	**Thematic Sessions**	**Thematic Sessions**	**Thematic Sessions**	
			19h - Event Dinner		

The XVISummer Workshop in Mathematics

Map of the University of Brasília

To access the university map, scan the following QR Code or click on the link:
`https://noticias.unb.br/images/mapas_campi_unb.pdf`

Plenary Speakers

Tarefa de Aprendizagem Profissional como um potente artefato para a formação de professores de matemática

Alessandro Jacques Ribeiro
UFABC, Santo André, Brasil

Abstract

Resultados de pesquisas apontam para a relevância e a contemporaneidade de se buscar identificar e compreender como decorre a aprendizagem profissional do professor de matemática, em particular ao longo de sua formação inicial. Assim, nosso grupo de pesquisa "ForMatE – Formação Matemática para o Ensino" tem realizado estudos que problematizam o uso de tarefas de aprendizagem profissional como um artefato que potencializa a exploração e articulação entre a matemática e a didática, e permite aproximar a universidade e a escola. Os resultados que serão apresentados e discutidos nesta conferência plenária relacionam-se com e são advindos de diferentes projetos de pesquisa realizados pelo grupo ForMatE e, particularmente, em um estudo liderado pelo professor Alessandro Jacques Ribeiro, por meio de uma bolsa de produtividade em pesquisa do CNPq, em vigência no período de 03/2023 a 02/2025. Considerando-se o Design-Basead Research como a metodologia dos projetos realizados pelo grupo, dos resultados referentes aos primeiros ciclos decorridos nos últimos anos, podemos destacar que os professores e futuros professores vêm (i) reorganizando seus conhecimentos matemáticos, com especial foco em conceitos algébricos; (ii) aprofundando sua reflexão sobre as dificuldades que os alunos da escola de ensino básico encontram e, com isso, ampliando seus conhecimentos didáticos sobre recursos e estratégias de ensino para superar essas dificuldades; (iii) tomando consciência de suas próprias dificuldades, seja acerca da matemática per se, seja no que refere aos processos de ensino e aprendizagem dela, o que os leva a superar tais dificuldades ao longo da participação nos processos formativos. Há também de se destacar como os formadores dos (futuros) professores também vivenciam oportunidades de aprender, em especial no que refere à ampliação de seus conhecimentos profissionais relacionados à matemática escolar e à dinâmica do trabalho docente nesse nível de ensino. Ao longo da conferência serão explorados exemplos das aprendizagens dos (futuros) professores e de seus formadores, bem como, dar-se-á destaque às contribuições das pesquisas realizadas pelo grupo no que refere (a) ao fortalecimento da presença e da interlocução da prática como um componente essencial no e para o conhecimento matemático e didático dos (futuros) professores para o ensino na escola básica; e (b) a repensar abordagens matemáticas e didáticas para os cursos de formação de professores, tanto nas escolas como nas universidades.

Trace monoids in 1-relator groups

Ashot Minasyan
University of Southampton, Southampton, England

Abstract

Given a finite simplicial graph Γ the trace monoid (a.k.a. partially commutative monoid) $T(\Gamma)$ associated to this graph is the monoid generated by the vertices of Γ subject to the relations that two vertices commute if and only if they are adjacent in Γ. A group with the same presentation is called the right angled Artin group $A(\Gamma)$. It is known that $A(\Gamma)$ contains $T(\Gamma)$ as its submonoid of positive words. Trace monoids originated in Computer Science, but more recently they have been used to establish certain undecidability results for 1-relation inverse monoids and groups. On the other hand, right angled Artin groups play an important role in Geometric Group Theory. In a recent work, Foniqi, Gray and Nyberg-Brodda showed that groups containing $T(P_4)$, where P_4 is the path with 4 vertices (of length 3), have undecidable rational subset problem. They also exhibited 1-relator groups containing $A(P_4)$ and asked whether every 1-relator group which has a submonoid isomorphic to $T(P_4)$ must also have a subgroup isomorphic to $A(P_4)$. In my talk I will discuss joint work with Motiejus Valiunas (University of Wroclaw, Poland) showing that the answer to the latter question is positive.

Theory and applications of aggregation functions

Benjamín René Callejas Bedregal
UFRN, Natal, Brasil

Abstract

The process of combining several numerical values into a single value that somehow represents all of them is called aggregation and the numerical function that carries out this process is called the aggregation function. In the context of fuzzy logic, aggregation functions are always increasing and preserve boundaries, and play an important role in fuzzy logic applications. In this talk we will present theoretical and practical aspects of aggregation functions in computer science, such as fuzzy formal languages, data classification and digital image processing.

Anti-unification: Introduction, Applications, and Recent Results

David M. Cerna
Czech Academy of Sciences, Prague, Czech Republic

Abstract

Anti-unification is a method for symbolically generalizing formal expression. It was introduced independently by Plotkin and Reynolds as an operation for inductive inferencing. Though conceptually simple, it is an effective tool for abstraction and templating. Since the seminal work, the number of applications has grown tremendously with uses in program analysis, program repair, library compression, automated reasoning, and beyond. With the growth of applications, there has been an effort to strengthen the theoretical foundations of the subject. In this talk, we introduce anti-unification, overview the existing applications, and discuss recent theoretical results concerning equational and high-order anti-unification.

Commuting probability for subgroups in finite group

Eloisa Detomi
Universidade de Padova, Padova, Italy

Abstract

In this talk I will discuss some problems concerning commuting probability of subgroups in finite groups. The commuting probability of a finite group G is defined to be the probability that two randomly chosen group elements commute. Following a review of some classical results on the commuting probability of G, we will discuss the behaviour of commuting probability when considering relevant subgroups of G, particularly focusing on Sylow subgroups.

Multiscale domain decomposition methods for large-scale computation of flows in heterogeneous porous media

Fabricio Simeoni de Sousa
ICMC/USP, São Carlos, Brasil

Abstract

The Multiscale Robin Couple Method (MRCM) is a multiscale domain decomposition method based on a mixed finite element formulation that allows for efficient solutions of second-order elliptic equations in a coarse scale, incorporating fine grid details of the solution through the efficient parallel computation of independent multiscale basis functions. The coupling between subdomains is performed by imposing Robin-type boundary conditions in the computation of the multiscale basis functions, ensuring that compatibility conditions are enforced on a large scale. This procedure generalizes the discrete version of well-known multiscale mixed methods, such as the Multiscale Mortar Mixed Finite Element Method (MMMFEM), the Multiscale Hybrid-Mixed Method (MHM), and the Multiscale Mixed Method (MuMM), via the suitable choice of the Robin boundary parameter and interface spaces. While generalizing those methods, it also introduces the possibility of adaptivity, resulting in accurate solutions compared to the undecomposed fine grid solution and other multiscale procedures based on the lowest order Raviart-Thomas finite element spaces. We will present the latest developments of the MRCM, in terms of interface enrichment and adaptivity, focusing on the accuracy of the decomposition, applications to complex porous media flow models, preconditioning, and speedup results on high-performance computations involving billions of fine grid cells.

On the tradeo between rates of almost sure convergence and overshoot integrability

Michael A. Högele
Universidad de los Andes, Bogotá, Colombia

Abstract

In this talk we start with an elementary, but useful, quantitative generalization of the first Borel-Cantelli lemma. The idea is to translate good rate of convergence of probability events into higher order moments of the overlap statistics. That is, it can quantify almost sure convergence in terms of the number of occurrences of the error events, which appear in the convergence in probability. We provide a sample of applications, such as the strong law of large numbers, the presence of a large deviations principle, the method of moments in statistics and numerical analysis. In the end we present results on martingale convergence, such as the convergence of Polya urns. This is joint work with Luisa F. Estrada and A. Steinicke.

A quick tour of variational methods in nonlinear PDE

Mónica Clapp
Universidad Nacional Autónoma de México, Cidade do México, México

Abstract

Many phenomena in physics, engineering, biology, economics, finance, and mathematics itself are described by nonlinear differential equations. A very important class of these, due to the great variety of phenomena they model, are the Euler-Lagrange equations, whose solutions satisfy an optmization criterion, generally given by an integral that represents a certain energy, an action, a cost function, etcs This type of equations are called variational problems. In a quick tour we will show how these problems are addressed and some of the challenges they present.

Minimal surfaces and eigenvalue problems

Vanderson Lima
UFRGS, Porto Alegre, Brasil

Abstract

In this talk I will survey some results exploring the connection between minimal surfaces, which are critical points of the area functional, and eigenvalues of certain self-adjoint operators. The focus will be on how this can be used to obtain characterization and existence results for minimal surfaces. In particular, I will present my own contribution to this topic, in joint work with Ana Menezes.

Special Sessions

Algebra

In memoriam to Rudolf Richard Maier

Central polynomials of matrix algebras

Ana Cristina Vieira
UFMG, Belo Horizonte, Brasil
This is a joint work with J. Cruz (UFMG)

Abstract

A polynomial $f = f(x_1, \cdots, x_n)$ in the free algebra $F\langle X\rangle$ is a central polynomial of an algebra A if it has zero constant term and $f(a_1, \cdots, a_n)$ belongs to the center of A, for all $a_1, \cdots, a_n \in A$. The description of the generators of the space of central polynomials $C(M_n(F))$, for $n \geq 2$, is known only in case $n = 2$ [Okhitin, 1988]. In the last years, the interest turned to matrix algebras endowed with additional structures, such as gradings and involutions. For these cases, Brandão and Koshlukov (2007) described the space of graded central polynomials of $M_2(F)$ with non-trivial grading and also the space of central $*$-polynomials of $M_2(F)$ with transpose and symplectic involutions. In this talk I will present some results about the space of central polynomials of graded matrix algebras endowed with an involution which preserves the grading.

Reminiscing about some of the most important results of Rudolf Maier's works

Angel Carocca
Universidad de La Frontera, Temuco, Chile

Abstract

The purpose of this talk is to recall and comment some of the most outstanding results obtained by R. Maier, which have inspired other researchers in their studies on permutability and subnormality of subgroups of finite groups.

he BNS invariants for surface braid groups

Carolina Miranda
UFES, Vitória, Brasil

Abstract

The Bieri-Neumann-Strebel invariant $\Sigma^1(G)$ of a finitely generated group G was first defined in 1987 [**?**], and it well known as an important object in Geometric Group Theory. Despite of its importance, there are only a few classes of groups for which Σ^1 is already known. In 2015, N. Koban, J. McCammond and J. Meier obtain the BNS invariant for the pure braid groups of the disc [**?**]. We compute and explicitly describe Σ^1 for the braid groups and the pure braid groups of some surfaces, namely, the sphere, the projective plane, the torus and the Klein bottle [**?**]. And, as further work, we are interrested to know if we are able to compute the BNS invariant for others closed surfaces or punctured surfaces. Joint work with Wagner Sgobbi.

Colorings of Latin squares and diagonal graphs

Csaba Schneider
UFMG, Belo Horizonte, Brasil

Abstract

I'll review some recent results and long standing conjectures concerning the colorings of Latin squares. A Latin square of order n requires at least n colors. The colorability with precisely n colors is equivalent to the existence of an orthogonal mate. Problems related to orthogonal mates were first studied by Euler and some of his questions were answered in the 20th and 21st century. Not all Latin squares, including Cayley tables of finite groups, possess orthogonal mates and the question of classifying finite groups whose Latin squares have orthogonal mates requires the Finite Simple Group Classification. I will also generalize the problem to higher-dimensional objects and present some results related to colorings of diagonal graphs. Some of these results have deep consequences in permutation group theory.

Homological Properties of Metabelian Restricted Lie Algebras

Esteban de Jesus
Unicamp, Campinas, Brasil

Abstract

This is a joint work with my supervisor D. H. Kochloukova. Recently, in a published paper in journal of Algebra, Kochloukova and Leon charaterised the finitely presented split metabelian restricted Lie algebras over a perfect fiel k. Our first result generalizes their result by dropping the assumption that the field k is perfect and that the extension is split. Our second result is the classification of split extension metabelian restricted Lie algebras (let us say L) of homological type FP_m i. e. the trivial module k over $calU_{res}(\mathcal{L})$ has a projective resolution with all module finitely generated in dimensions up to m. The same problems for ordinary Lie algebras were treated ealier by Bryant and Groves (in the case of finite presentability) and by Kochloukova (for the type FP_m).

Groups with a solvable subgroup of prime-power index

Raimundo Bastos
UnB, Brasília, Brasil

Abstract

I will present some results that were obtained in collaboration with Csaba Schneider (Universidade Federal de Minas Gerais) concerning groups that have a solvable subgroup of prime-power index. Under weak conditions, such groups are solvable and, when they are not, the index of their solvable radical is asymptotically small.

Self-similarity of finitely-generated torsion-free metabelian groups

Túlio
UFRJ, Rio de Janeiro, Brasil

Abstract

In this presentation, we will discuss results obtained in collaboration with A. C. Dantas, A. A. Berlatto, and S. N. Sidki. A group G is self-similar if it acts on a one-rooted m-regular tree $\mathcal{T}_m$, where the states of its elements are elements of G. Additionally, G is transitive self-similar if it induces a transitive action on the first level of the tree. If a self-similar group G does not admit a transitive action, we refer to it as an intransitive self-similar group. Let A be a finitely generated abelian group. We demonstrate that the group $G = A \wr \mathbb{Z}^d$ exhibits an intransitive self-similar action in some tree $\mathcal{T}_m$. In particular, the group $\mathbb{Z}^l \wr \mathbb{Z}^d$ is an intransitive self-similar group. We establish that any 2-generated torsion-free nilpotent group of class 3 can be faithfully represented as a transitive self-similar group. Moreover, we provide an example of a 4-generated torsion-free nilpotent group of class 3 that does not admit such a representation. This particular example is derived from the work of V. Bludov and B. Gusev, leading us to the conclusion that the minimal class c for which examples of non-self-similar finitely-generated torsion-free nilpotent groups exist is 3.

Analysis

Estimate for concentration level of the Adams functional and extremals for Adams-type inequality

Abiel Costa Macedo
UFG, Goiânia, Brasil

Abstract

TBA

A bit more of the classical Brezis-Nirenberg problem

Adilson E. Presoto
UFSCar, São Carlos, Brasil

Abstract

In the well-known paper [**?**], Brezis and Nirenberg have proved that if $0 < \lambda < \lambda_1$ and $N \geq 4$, then

$$\begin{cases} -\Delta u = \lambda u + |u|^{2^*-2}u \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{P}$$

where $2^* = 2N/(N-2)$ and λ is a real parameter, has a positive solution. In the following years, Capozzi, Fortunato and Palmieri extended the result for all $\lambda > 0$ if $N \geq 5$ or $\lambda > 0$ and $\lambda \neq \lambda_k$ in the case $N = 4$. Here $0 < \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_j \leq \cdots$ denote the eigenvalues of $-\Delta$ in $H_0^1(\Omega)$. In this talk, we will deal with the resonant case, i.e. when λ an eigenvalue, in 4 dimensions. In this situation, a solution can no longer be obtained by applying the usual linking theorem. In , the authors use a version of the symmetric mountain pass theorem to solve the problem for $N \geq 5$ and λ being an eigenvalue, due to technicalities the proof does not work if $N = 4$. It seems to us that the main difficult to apply a minimax theorem is to gettting the right estimates. For instance, we are not able to get a separation condition between the values over the boundary of a rectangle and a sphere in the finite-dimension subspace in order to apply the classical linking theorem. To overcome this, we use a variant of the linking theorem, where this condition is relaxed. We point out that this generalization was motivated by the orthogonalization technique introduced by Gazzolla and Ruf, and some variant of the classical minimax theorems without the separation condition obtained by Silva.

Some results on normalized solutions for the Schrödinger equations

Chao Ji
University of Science and Technology, Langfang, China
These are joint works with C.O. Alves and O. H. Miyagaki

Abstract

In this talk, we will introduce some results on normalised solutions for the Schörodinger equations. First of all, we show the existence of normalised solutions for the Schrödinger equations with L^2-subcritical growth and different types of potentials. Then, existence and multiplicity of normalized solutions for a Schrodinger equation with critical growth in RN will be given. Finally, we investigate the existence of multiple normalized solutions to the logarithmic Schrodinger equation via the minimization techniques and the Lusternik-Schnirelmann category.

PROBLEMS OF CONCAVE AND CONVEX TYPE IN $\mathbb{R}^n$ WITH BOUNDED WEIGHT

Denilson Pereira
UFCG, Campina Grande, Brasil
Joint work with Pedro Ubilla (USACH), Juan Arratia(USACH) and Diego Ferraz (UFRN)

Abstract

In this talk we present recent results involving elliptic problems with sub-linear growth at the origin and super-linear behaviour at infinity with bounded weights in $\mathbb{R}^n$. Problems of this nature present several difficulties to deal with variational, once that the energy functional is not, a priori, well defined. We overcome this difficulty by obtaining a first solution, via the sub and super solution technique. Understanding the behaviour at infinity of this first solution, we are able to obtain a second solution via variational methods.

Schrödinger-Poisson system with zero mass in $\mathbb{R}^2$ involving (2, q)-Laplacian: existence, asymptotic behaviour and regularity of solutions

Edcarlos D. Silva
UFG, Goiânia, Brasil
This is a join work with J. C. Albuquerque - UFPE and J. Carvalho - UFS

Abstract

It is stablished the existence of positive least energy solution for the following class of planar ellipitic systems in the zero mass case

$$\begin{aligned} -\Delta u - \Delta_q u + \phi|u|^{r-2}u = \lambda|u|^{p-2}u, \quad &\text{in } \mathbb{R}^2, \\ \Delta\phi = 2\pi|u|^r, \quad &\text{in } \mathbb{R}^2, \end{aligned}$$

where $\lambda \leq 0$, $1 < q < 2$, $q^* := 2q/(2-q) < r < \infty$ and $p \geq 2r$. Due to the nature of the problem, we deal with the logarithmic integral kernel. Our approach is based on Nehari manifold and a version of the Principle of Symmetric Criticality due to Palais. Futermore, we study the asymptotic behavior of the solutions wherenever the parameter λ goes to zero or infinity. Finally, we study regularity of the solutions applying Moser interation scheme.

Critical points with prescribed energy for a class of functionals depending on a parameter

Humberto Quoirin
Universidad Nacional de Córdoba, Córdoba, Argentina

Abstract

Given $c \in \mathbb{R}$ we look for couples $(\lambda, \mu) \in \mathbb{R} \times X$ solving the problem

$$\phi'_\lambda(u) = 0,\ \phi_\lambda(u) = c$$

Here $\phi_\lambda = I_1 - \lambda_2$, , where I_1 and I_2 are C^1 even functionals on a Banach space X. Under further conditions on I_1 andI_2 we prove the existence of infinitely many couples $(\lambda_{n,c}, u_{n,c})$ solving this problem. More generally, we analyze the structure of the solution set of this problem with respect to λ and c. In particular, we show that the maps $c \mapsto \lambda_{n,c}$ are continuous, which gives rise to a family of *energy curves* for this problem. The analysis of these curves provide us with several bifurcation type results, which are then applied to some elliptic problems. Our approach is based on the *nonlinear generalized Rayleigh quotient* method.

p-Harmonic functions in the upper half-space

João Marcos do Ó
UFPB, João Pessoa, Brasil

Abstract

We investigate the existence, nonexistence, and qualitative properties of p-harmonic functions in the upper half-space $\mathbb{R}^N_+$ $(N \geq 3)$ satisfying nonlinear boundary conditions. Moreover, the symmetry of positive solutions is shown by using the method of moving planes.

The method of the Nehari Manifold on cones

João R. Santos Junior
UFPA, Belém, Brasil

Abstract

TBA

Sharp regularity for the obstacle problem for p-Laplacian type equations and applications

João Vitor da Silva
Unicamp, Campinas, Brasil
This is a joint work with Elzon C. Bezerra Júnior (UFCA) and Romário T. Frias (Unicamp)

Abstract

In this Lecture we show existence/uniqueness of weak solutions of an obstacle problem for a quasi-linear operator with unbounded source terms. In our results, we obtain sharp gradient estimates, namely, $C^{1,\alpha}_{loc}(B_1)$ for the solution to an explicit and universal regularity exponent. Our results are relevant even for the simplest model case governed by the p-Laplacian with Hölder continuous coeficients

$$\begin{aligned} div(|\nabla_u|^{p-2}\mathfrak{U})\nabla_u = f(x), \quad &\text{in } \{u > \varphi\} \cap B_1 \\ div(|\nabla_u|^{p-2}\mathfrak{U}(x)\nabla_u \leq f(x), \quad &\text{in } B_1 \\ u(x) \geq \varphi(x), \quad &\text{in } B_1 \\ u(x) = \varphi(x), \quad &\text{on } \partial B_1, \end{aligned}$$

where $f \in L^q(\Omega)$ for $q > n$ and $q \geq \frac{p}{p-1}$ $(1 < p < \infty), \mathfrak{U} \in C^{0,\sigma}(\Omega, \mathbb{R}^{n\times n})$ (for some $\sigma \in (0,1]$) with $\mathfrak{U}$ a (λ, Λ)-uniformly elliptic matrix, and $\varphi \in C^{1,\beta}(\Omega) \cap \mathfrak{X}_{p,q}$, for some $\sigma \in (0,1]$) with $\mathfrak{U}$ a (λ, Λ)- uniformly elliptic matrix, and $\varphi \in C^{1,\beta}(\Omega) \cap \mathfrak{X}_{p,q}$, for some $\beta \in (0,1]$ where

$$\mathfrak{X}_{p,q} := \{v \in W^{1,p}(\Omega); \quad div\ \mathfrak{a}(x, \nabla v) \in L^q(\Omega)\}.$$

For some specic scenarios, we show the non-degeneracy of solutions, which provides crucial information about the free boundary of solutions. Our regularity estimates improve and extend, to a certain extent, results previously obtained for the obstacle problem governed by the p-Laplacian with bounded source term (cf. [1] and [3]). Furthermore, we gave special emphasis to the study of the linear and non-homogeneous case, i.e., $p = 2$ and $f \neq 0$, which was not available in the literature and it plays a decisive role in analysing the non-linear case (cf. [2]).

On the structure of the Nehari set associated to a Schrodinger-Poisson system with prescribed mass: old and new results

Kaye Silva
UFG, Goiânia, pais

Abstract

Consider the Schrödinger-Poisson system, with prescribed L^2 norm, in the whole $mathbbR^3$

$$-\Delta u + q\phi_u u - \lambda|u|^{p-2}u = llu, \quad \text{in } \mathbb{R}^3$$
$$\int u^2 = r$$

By using the fibering method of Pohozaev and the notion of Extremal Parameters introduced by Il'yasov we show that many results in the literature concerning this system can be obtained in an unified way. We exhibit also new results.

On the solvability with borderline regularity for the 2D inviscid Boussinesq equations

Lucas C. F. Ferreira
Unicamp, Campinas, Brasil

Abstract

We are concerned with the long-time solvability for 2D inviscid Boussinesq equations for a larger class of initial data which covers the case of borderline regularity. First we show the local solvability in Besov spaces uniformly with respect to a parameter κ associated with the stratification of the fluid. Afterwards, employing a blow-up criterion and Strichartz-type estimates, the long-time solvability is obtained for large κ regardless of the size of initial data.

Comparison Theorem for the fractional Orlicz Sobolev space

Marcos L. M. Carvalho
UFG, Goiânia, pais

Abstract

In this work, we obtain a new version of the comparison principle for sub and super solutions to an equation involving the fractional ϕ-Laplacian operator. We consider a problem in a local sense, instead of zero-boundary conditions.

Boundary Weak Harnack Estimates and Regularity for Elliptic Operators in Divergence Form and Applications in PDEs

Mayra Soares
UnB, Brasília, Brasil
Joint work with Boyan Sirakov and Fiorella Rendón

Abstract

We obtain a global extension of the classical Weak Harnack Inequality which extends and quanties the HopfOleinik boundary-point lemma, for uniformly elliptic equations in divergence form, under the weakest assumptions on the leading coecients and on the boundary of the domain. Our main tool is the use of suitable barrier functions, which are solutions of auxiliaries problems and the C^1-estimates up to the boundary. Among the consequences is a boundary gradient estimate, due to Krylov and well-studied for non-divergence form equations, but completely novel in the divergence framework. Another consequence is a new more general version of the HopfOleinik lemma. Furthermore, we provide an application showing how to use this results in order to deduce a priori upper bounds and multiplicity of solutions for a class of quasilinear elliptic problems with quadratic growth on the gradient.

Geometry

Enneper representation of minimal surfaces in the three-dimensional Lorentz–Minkowski space

Adriana Cintra
UFG, Goiânia, Brasil

Abstract

The Weierstrass representation formula for minimal surfaces in $\mathbb{R}^3$ is a powerful tool to construct examples and to prove general properties of such surfaces, since it gives a parametrization of minimal surfaces by holomorphic data. In [**?**] the authors described a general Weierstrass representation formula for simply connected immersed minimal surfaces in an arbitrary Riemannian manifold. In [**?**], Andrade introduces a new method to obtain minimal surfaces in the Euclidean 3-space which is equivalent to the classical Weierstrass representation and, also, he proves that any immersed minimal surfaces in $\mathbb{R}^3$ can be obtained using it. This method has the advantage of computational simplicity, with respect to the Weierstrass representation formula and it allows to construct a conformal minimal immersion $\psi : \Omega \subset \mathbb{C} \to \mathbb{C} \times \mathbb{R}$, from a harmonic function $h : \Omega \to \mathbb{R}$, provided that we choose holomorphic complex valued functions L, P on the simply connected domain Ω such that $L_z\, P_z = (h_z)^2$. The immersion results in $\psi(z) = (L(z) - \overline{P(z)}, h(z))$ and it is called *Enneper immersion* associated to h. Besides, the image $\psi(\Omega)$ is called an *Enneper graph* of h. Some extensions of the Enneper-type representation in others ambient spaces have been given in [**?**] and [**?**]. The aim of the paper is to illustrate an Enneper-type representation for timelike minimal surfaces in the Lorentz-Minkowski space $\mathbb{L}^3$ (veja [**?**]).

Submanifolds od Constant Mean Curvature: rigidity and stability

Adriano Bezerra
IFGo, Trindade, Brasil

Abstract

In this talk we will establish conditions on the first eigenvalue of the stability operator and on the length of the traceless part of the second fundamental form of a complete submanifold M^n with constant mean curvature in $M^{n+m}(c)$ to show that M^n has some geometric property of rigidity.

A proof for Besse's conjecture

Benedito Leandro
UnB, Brasília, Brasil

Abstract

This study investigates the topological implications arising from stable (free boundary) minimal surfaces in a static perfect fluid space while ensuring that the fluid satisfies certain energy conditions. Based on the main findings, it has been established the topology of the level set $\{f = c\}$ (the boundary of a stellar model), where c is a positive constant and f is the static potential of a static perfect fluid space. We prove a non-existence result of stable free boundary minimal surfaces in a static perfect fluid space. An upper bound for the Hawking mass for the level set $\{f = c\}$ in a non-compact static perfect fluid space was derived, and the positivity of Hawking mass is provided in the compact case when the boundary $\{f = c\}$ is a topological sphere. We dedicate a section to revisit the Tolman-Oppenheimer-Volkoff solution, an important procedure for producing static stellar models. We will present a new static stellar model inspired by Witten's black hole (or Hamilton's cigar).

Einstein submanifolds with parallel mean curvature in product spaces

Fernando Manfio
USP, São Paulo, Brasil
This is a joint work with Estela Garcia

Abstract

In this work, we provide a classification of Einstein submanifolds in product spaces $\mathbb{Q}_\epsilon^n \times \mathbb{R}$, with flat normal bundle and parallel mean curvature vector field. This extends a previous result due to Leandro, Pina and dos Santos (Bull. Braz. Math. Soc., 2020) for Einstein hypersurfaces in such ambient space.

Sharp lower bound for the charged Hawking mass in the electrostatic space

Guilherme Sabo
UFG, Goiânia, Brasil

Abstract

We prove a sharp lower bound for the charged Hawking mass of a stable minimal two-sphere in a three-dimensional electrostatic space. An upper bound for the genus of a stable CMC surface in the electrostatic system is provided. We also study the positivity for the charged Hawking mass of a minimal surface with index one and a stable CMC surface in the electrostatic space. A criterion for a CMC surface in the Reissner-Nordstrom deSitter space to be stable is presented.

Sólitons de Rotação do Fluxo de Curvatura Média na Esfera de Berger

Hiuri Reis
UFG, Goiânia, Brasil

Abstract

O Fluxo de Curvatura Média (FCM) é um fluxo do tipo gradiente para o funcional volume . No âmbito do FCM, uma hipersuperfície M^{n+1} evolui localmente na direção onde o elemento de volume diminui mais rapidamente e eventualmente se torna extinto. Ao longo do fluxo podem ocorrer singularidades e há grande interesse em seu estudo. Há uma extensa literatura sobre o assunto, remetemos o leitor ao excelente levantamento de Colding, Minicozzie Pedersen [2] e às referências nele contidas. Nos últimos anos, soluções auto-similares, dadas pela composição de isometrias e homotetias, tem sido intensamente estudadas. No espaço Euclidiano, os exemplos mais simples de soluções auto-similares são as esferas e os cilindros, que são hipersuperfícies por contração. Exemplos de hipersuperfícies por translação são encontradas em [1]. Existem poucos resultados sobre o fluxo de curvatura média em espaços não Euclidianos. Em [4], Hungerbhler e Smoczyk consideraram um caso particular de soluções auto-similares evoluindo pelo FCM por um grupo de isometrias do espaço ambiente, que são conhecidas como sólitons, e apresentaram vários exemplos dessas hipersuperfícies em variedades Riemannianas. Neste trabalho estudamos as soluções sóliton, soluções auto-similares que evoluem por isometrias, na Esfera de Berger. Caracterizamos com um sistema de equações diferenciais ordinária as soluções sólitos do FCM que são superfícies de rotação imersas na Esfera de Berge [3]. Fazendo uma análise qualitativa dessas equações, mostramos que os dois fins dessas superícies assintótam um toro flat.

Constant mean curvature hypersurface in $H^{n+1}(-1)$ and space-like hypersurfaces in M^{n+1}

Hudson Pina
UFMT, Cuiabá, Brasil

Abstract

Let M be a hypersurface with constant mean curvature (CMC hypersurfaces) immersed in a hyperbolic space $\mathbb{H}^{n+1}(-1)$ or a space-like hypersurface with constant mean curvature immersed in a Lorentz space form $M_1^{n+1}(c)$, $c \in \{-1, 0, 1\}$ such that the $\mathbb{L}^d$ norm of $|\phi|$, for some d, on geodesic balls centered at some point $p \in M$ has less than quadratic growth, where, ϕ is the traceless part of the second fundamental form of M. We find additional conditions to imply that M is totally umbilical.

Deformations of the σ_2-curvature and volume

Maria Andrade
UFS, Aracaju, Brasil
This is part of a joint work with A. Silva Santos (UFS) and T. Cruz (UFAL)

Abstract

In this talk, we show some results about deformations of the σ_2-curvature and volume. In particular, for closed manifolds, we relate critical points of the total σ_2-curvature functional to σ_2-Einstein metrics.

Geometry of static perfect fluid space-time

Neilha Pinheiro
UFAM, Manaus, Brasil
This is a joint work with J. Costa, R. Diógenes and E. Ribeiro Jr.

Abstract

In this talk, we discuss the geometry of static perfect fluid space-time (SPFST) on compact manifolds with boundary. We use the generalized Reilly's formula to establish a geometric inequality for a SPFST involving the area of the boundary and its volume. Moreover, we obtain new boundary estimates for SPFST. One of the boundary estimates is obtained in terms of the Brown–York mass and another one related to the first eigenvalue of the Jacobi operator. In addition, we provide a new (simply connected) counterexample to the Cosmic no-hair conjecture for arbitrary dimension greater than or equal four.

Three–dimensional conformally flat electrostatic system

Róbson Lousa
UFRR, Amajari, Brasil

Abstract

Einstein Field Equation coupled to Maxwell Fields is called Electrostatic System. Three-dimensional electrostatic manifolds with divergence-free Bach tensor are locally conformally flat, provided that the electric field and the gradient of the lapse function are linearly dependent. Consequently, a three-dimensional electrostatic manifold admits a local warped product structure with a one-dimensional base and a constant curvature surface fiber.

On Stability and Isoperimetry of Constant Mean Curvature Spheres of Hn × R and Sn × R

Ronaldo Freire de Lima
UFRN, Natal, Brasil

Abstract

In this talk, which is based on a joint work with M. F. Elbert (UFRJ) and B. Nelli (Universitá di L'Aquila), we approach the one-parameter family of rotational constant mean curvature (CMC) spheres of $\mathbb{H}^n \times \mathbb{R}$ and $\mathbb{S}^n \times \mathbb{R}$, focusing on their stability and isoperimetry properties. Our results include the proof of the uniqueness of the regions enclosed by the rotational CMC spheres of $\mathbb{H}^n \times \mathbb{R}$ as solutions to the isoperimetric problem, which fills in a gap in the original proof given by Hsiang and Hsiang. We also establish that all CMC spheres of $\mathbb{H}^n \times \mathbb{R}$ are stable, and so are those of $\mathbb{S}^n \times \mathbb{R}$ with sufficiently large mean curvature. In addition, we show that there exists a one-parameter family of CMC spheres in $\mathbb{S}^n \times \mathbb{R}$ which are stable and non-isoperimetric (i.e., they do not bound isoperimetric regions). In presenting these results, we intend to make clear that, in essence, they come from the fact that the rotational CMC spheres of $\mathbb{H}^n \times \mathbb{R}$, and those of $\mathbb{S}^n \times \mathbb{R}$ with sufficiently large mean curvature, are nested.

Ends of ρ-Einstein solitons

Valter Borges
UFPA, Belém, Brasil

Abstract

In this talk we show shrinking ρ-Einstein solitons are connected at infinity, if $\rho \in [0, 1/2(n-1))$ and provided we assume a certain bound on the scalar curvature. We also discuss the ends structure when $\rho = 1/2(n-1)$. These results were obtained in joint works with H. A. Rosero-García - UnB and J. P. dos Santos - UnB.

Logic and Computation

On the Nominal Semantics of Nominal Algebra with Fixed-Point Constraints

Ali Khăn Caires Ribeiro Santos
UnB, Brasília, Brasil

Abstract

Nominal algebra is a framework for interpreting and reasoning about systems with binding, using nominal terms as its term language. Its derivation rules are subject to freshness conditions and extend first-order terms with object-level variables, meta-level variables, and constructs for binding, alphaequivalence, and capture avoidance. The semantics of nominal algebra are in the class of nominal sets, which have a rich structure for involving names, permutation, and name binding. Previous work has proven the soundness and completeness of nominal algebra, with applications including interpreting solutions to nominal (dis)unification problems. However, recent investigations show that the intentional semantics of nominal algebra with fixed-point constraints is not sound. This talk presents a counterexample and proposes solutions by exploring a subclass of nominal sets called strong nominal sets and necessary modifications in derivation rules.

Anti-Unification on Absorption Theories

Andrés Felipe González Barragán
UnB, Brasília, Brasil

Abstract

Anti-unification or generalization is the process of finding commonalities between expressions. There are many applications of generalization as clone and plagiarism detection. While syntactic forms of anti-unification are enough for many applications, some aspects of software analysis methods are more appropriately modeled by reasoning modulo equational theories. Some important theories include the absorption property; i.e., operators with axioms $f(x, \epsilon f) \approx f(\epsilon f, x) \approx \epsilon f$. This work presents a sound and complete anti- unification algorithm for such theories. Additionally, it shows that anti- unification of absorption theories is of type infinitary, and provides a finitary representation of the minimal complete set of generalizations.

Formalising Combinatorial Mathematics: A Modular Approach

Chelsea Edmonds
University of Sheffield, Sheffield, England

Abstract

Formalised mathematics has historically focussed on the goal of formalising specific well-known theorems, which can lead to issues around reusability and accessibility, especially as formal libraries continue to grow at a rapid pace. My work on formalising combinatorics in Isabelle/HOL presents an approach which places the primary focus on the formalisation of proof techniques and underlying structures, making use of locales, Isabelle's module system. In this talk, I'll give an introduction to the locale-centric methodology for formalising mathematical structures to create flexible, modular and extensible libraries. Additionally, I'll demonstrate how the same locale-centric methods can be applied to set up proof contexts to support general proof techniques. I'll conclude by demonstrating an example probabilistic technique to formalise some results on hypergraphs.

Building and Combining Unification and Matching Procedures

Christophe Ringeissen
Inria, Nancy, França

Abstract

The concept of unification is ubiquitous in logic programming and in automated reasoning, for instance to perform deduction in theorem proving. Equational unification consists in finding instances of terms so that these instances are equivalent with respect to an equational theory given by some set of axioms. Equational unification being undecidable in general, it is important to identify equational theories and simple cases where it is possible to obtain sound, complete and terminating unification procedures. Matching is a particular (simple) case of unification of greatest interest to execute rule-based programs and to perform simplification in theorem proving. We show a systematic approach to build and to combine unification and matching algorithms for a large class of equational theories. We illustrate the approach on equational theories of practical interest in the analysis of security protocols.

Quantitative Weak Linearisation

Daniel Lima Ventura
UFG, Goiânia, Brasil
This is a joint work with Patrick Baillot, Ugo Dal Lago, and Cynthia Kop.

Abstract

The class of basic feasible functionals (BFF) is the analog of PF (polynomial time functions) for type-two functionals, that is, functionals that can take (first-order) functions as arguments. BFF can be defined by means of oracle Turing machines of time bounded by a second-order polynomial. On the other hand, higher-order term rewriting provides an elegant formalism for expressing higher-order computation. In this talk, address the problem of characterizing the class BFF by higher-order term rewriting. Various kinds of interpretations for first-order term rewriting have been introduced in the literature for proving termination and characterizing (first-order) complexity classes. In this recent work, we consider a notion of cost-size interpretations for higher-order term rewriting and see definitions as ways of computing functionals. We then discuss a novel characterization of BFFs by means of higher-order rewriting and the higher-order interpretation method. Finally, we will shed light on the soundness and completeness of such rewriting-based characterizations of higher-order complexity classes.

Matching Plans for Frame Inference in Compositional Reasoning

Daniele Nantes-Sobrinho
UnB/Imperial College, Brasília/London, Brasil/England
Joint work with Andreas Lööw, Petar Maksimovic, Philippa Gardner, Sacha Ayoun

Abstract

The use of function specifications and manipulation of user- defined predicates are two essential ingredients of modern compositional verification tools. To execute these operations successfully, these tools must be able to solve the frame inference problem, that is, understand the parts of the state relevant for the operation at hand. We introduce matching plans, which facilitate frame inference and advance the state-of- the-art in that they allow tool users to write intuitive and understandable specifications and the tools to efficiently infer the information required to perform frame inference.

Closed Rewriting - Checking overlaps of Nominal Rewriting Rules

Daniella Santaguida Magalhães de Souza
UnB, Brasília, Brasil

Abstract

Nominal rewriting is not complete for equational reasoning in general; however, closed nominal rewriting is complete for equational reasoning with closed axioms [1]. Intuitively, no free atom occurs in a closed term, and, as a natural assumption, closed axioms do not allow abstracted atoms to become free. In this talk, we will analyze the confluence of nominal rewriting systems (NRS), we will check whether closedness can be useful/essencial to guarantee the confluence of a NRS, and finally we will make some observations on possible extensions for proving confluence modulo equational theories.

A New Characterisation of BFFs via Higher-Order Rewriting

Deivid Vale
Radboud University, Nijmegen, Netherlands
This is a joint work with Patrick Baillot, Ugo Dal Lago, and Cynthia Kop.

Abstract

The class of basic feasible functionals (BFF) is the analog of PF (polynomial time functions) for type-two functionals, that is, functionals that can take (first-order) functions as arguments. BFF can be defined by means of oracle Turing machines of time bounded by a second-order polynomial. On the other hand, higher-order term rewriting provides an elegant formalism for expressing higher-order computation. In this talk, address the problem of characterizing the class BFF by higher-order term rewriting. Various kinds of interpretations for first-order term rewriting have been introduced in the literature for proving termination and characterizing (first-order) complexity classes. In this recent work, we consider a notion of cost-size interpretations for higher-order term rewriting and see definitions as ways of computing functionals. We then discuss a novel characterization of BFFs by means of higher-order rewriting and the higher-order interpretation method. Finally, we will shed light on the soundness and completeness of such rewriting-based characterizations of higher-order complexity classes.

Propositional Proofs Compression

Edward Hermann Haeusler
PUC-Rio, Rio de Janeiro, Brasil

Abstract

This talk presents a novel compressing propositional proofs algorithm and discusses its relevance in solving some computational complexity well-known conjectures.

Anti-Unification on Terms With Different Types

Gabriela de Souza Ferreira
UnB, Brasília, Brasil

Abstract

Anti-unification is the problem of expressing the most common structure between two given expressions. The different problems of anti-unification depend on how such commonalities are established and structured. In this talk the focus is in the universe of Typed Lambda Terms, more precisely, in how to identify the common structures of expressions that may have different types. First, a fast presentation of the anti- unification problem for inputs with the same type will be given, with the propose of highlighting the difference between the standard problem and well-known more general problems. Secondly, some preliminar ideas about how to solve anti-unification for terms with different types will be discussed. The presentation will use several examples and illustrations.

Bounded ACh Unification

Guilherme Borges Brandão
UnB, Brasília, Brasil

Abstract

E-Unification is a procedure to find solutions to a set of equations between firstorder terms with respect to an equational theory E. ACh theory takes a function symbol h that is an homomorphism over an associative-commutative function symbol. Previous work proved that ACh Unification is an undecidable problem [1]. Recently, however, it was defined an approximation of the problem called Bounded ACh Unification [2], that consists in bounding the number of times that h is applied recursively. In this talk, we will discuss the inference rules for solving this problem and verify the proofs of termination, soundness and completeness.

Formalising local fields in the Lean theorem prover

María Inés de Frutos-Fernández
University of Sheffield, Sheffield, England
This is joint work with Filippo A. E. Nuccio.

Abstract

Local fields, and more generally fields complete with respect to a discrete valuation, are essential objects in commutative algebra, with applications to number theory and algebraic geometry. We formalize in Lean the basic theory of discretely valued fields, as well as the abstract definition and some fundamental properties of local fields.

Exemplifying Contemporary Formal Mathematics

Maurício Ayala-Rincón
UnB, Brasília, Brasil

Abstract

Computer-verified theories are the class of "complete" formalizations mathematicians have aimed for centuries. Theorem's proofs formalized in "proof assistants" provide certificates of their "correctness" and the required grade of granularity to be considered "complete." Such proof's mechanizations also come with tools to be ported by mathematicians and other professionals into robust technological tools independently of mathematical intuition, skills, hierarchy, or folklore. Computational formalizations provide absolutely correct proofs except for remote possibilities of computer failures. In this talk, we will select simple mathematical formalizations developed by the local group members to illustrate how such formalizations are developed in the proof assistant PVS.

Nikson Bernardes Fernandes Ferreira

How do PVS strategies help to prove correct float-point implementations?

Nikson Bernardes Fernandes Ferreira
UnB, Brasília, Brasil

Abstract

Numerical programs are usually taught using real arithmetic but implemented in floating-point arithmetic. The Floats domain does not inherit known Real properties such as distributivity and carries propagating representational errors that can lead to unexpected behaviors. It is hazardous for safety-critical applications such as air traffic management. Following a methodology introduced by M. Moscato, L. Titolo et al., given a logically verified PVS Real specification, it is possible to get a provably correct floating-point implementation, for which it is also possible to obtain mathematical certificates (proofs) ensuring its correct behavior. The proof structure of such certificates is suitable for being fully proved using PVS strategies. This talk will show how we develop PVS strategies to improve proofs' automation degree.

Mathematical Education

APRENDER MATEMÁTICA, FAZENDO MATEMÁTICA: PROPOSTA DE INTERVENÇÃO DIDÁTICA

Angel Homero Flores Samaniego
Universidad Nacional Autónoma do México, Cidade do México, México

Abstract

Independentemente das teorias e metodologias sobre como ensinar melhor a matemática na escola, os alunos ainda não a aprendem realmente e os professores ainda não sabem o que fazer para melhorar a aprendizagem dos seus alunos. Se refletirmos sobre a situação de outras disciplinas escolares e de outros ramos do conhecimento, a situação não é muito diferente. Os alunos têm dificuldade em aprender biologia ou química; o conhecimento adquirido no estudo da história é mínimo e se perde rapidamente. A utilização da língua materna é cada vez mais deficiente. As causas desta deterioração são múltiplas e têm origem, principalmente, na estrutura do sistema educativo e na educação informal que nossos alunos recebem através dos meios de comunicação. Revertê-lo não é fácil, porém, a partir da didática é possível fornecer elementos para melhorar a aprendizagem dos alunos e contribuir com um grão de areia. Aprender Matemática, Fazendo Matemática (AMHM, por suas siglas em espanhol), é um modelo de intervenção didática com o qual um grupo de professores mexicanos tenta melhorar a educação matemática de nossos alunos. Possui, entre outras, duas características que consideramos importantes: por uma parte, a separação da unidade de ensino-aprendizagem para dar ênfase à aprendizagem do aluno e não às estratégias de ensino do professor; e, por outra, o desenvolvimento conjunto do pensamento reflexivo dos alunos que, em nossa opinião, é a base de todo o conhecimento, científico e não científico. Nesta palestra darei mais detalhes sobre essas duas características, e como o conceito de avaliação em sala de aulas se conecta à melhoria da aprendizagem matemática dos alunos e é a base para fazer pesquisa educacional em sala de aulas.

A criatividade em matemática no contexto das altas habilidades ou superdotação em Matemática

Douglas Melo Fontes
UFAC, Rio Branco, Brasil

Abstract

A interseção entre criatividade em Matemática e altas habilidades ou superdotação no campo da Matemática é um campo instigante de pesquisa, que revela nuances importantes sobre o desenvolvimento cognitivo e educacional dos estudantes que apresentam essas características. Aponta- se que a escolha dessa temática para pesquisa está relacionada ao baixo números de estudantes identificados com altas habilidades ou superdotação e, ao mesmo tempo, à necessidade de destacar a relevância de reconhecer e fomentar o potencial excepcional em Matemática desses estudantes. Buscou- se, para estudar essa intersecção, um referencial teórico fundamentado no trabalho de Renzulli (1986), no que se diz respeito ao conceito de altas habilidades ou superdotação, destacando os processos de aprendizagens dos estudantes com tais características, enfatizando particularmente, a criatividade. Além disso, a pesquisa se apoia no estudo de Gontijo (2007), que trata do conceito de criatividade em Matemática e do uso de resolução de problemas. Para a compreensão da relação entre criatividade e altas habilidades ou superdotação em Matemática, estudos vêm sendo desenvolvidos a partir da aplicação de testes para medir a criatividade em Matemática (Fonseca, 2015; Gontijo, 2007), nos quais predominam situações envolvendo a resolução e a elaboração de problemas. Considerando esses elementos, serão apresentados resultados de um estudo baseado no desenvolvimento e aplicação de um teste de criatividade em Matemática (Fontes, 2022) a estudantes que são atendidos pelo Núcleo de Atividades em Altas Habilidades/ Superdotação – NAAH/S, no Estado do Acre. A pesquisa mostrou que o uso de itens de testes pode ser uma estratégia educacional promissora para estimular a criatividade desses estudantes. Destaca-se que indivíduos com altas habilidades ou superdotação em Matemática, frequentemente demonstram uma singular capacidade para abordar problemas de maneiras inovadoras e criativas. Sendo assim, a criatividade nesses estudantes pode se manifestar na formulação de soluções únicas, na aplicação de conceitos matemáticos em contextos não convencionais e na exploração de novas abordagens para desafios matemáticos. Por fim, destaca-se que a criatividade em Matemática desempenha um papel crucial no desenvolvimento de estudantes com altas habilidades ou superdotados nesta área, na busca de promover avanços ainda mais significativos no desenvolvimento de talentos em Matemática.

Proposição e Resolução de Problemas: pesquisa e prática

Flávia Sueli Fabiani Marcatto
Universidade Federal de Itajubá, Itajubá, Brasil

Abstract

A importância da Resolução de Problemas (RP), assim como, da Proposição de Problemas (PP) é reconhecida há muito tempo, porém a implementação nas salas de aula ainda é um desafio, especialmente na formação inicial de professores de matemática. Apesar de mais de 50 anos de pesquisa, a implementação da RP e da PP continua sendo um desafio para os professores (Chapman, 2016). Liljedahl e Cai (2022) defendem que para alguns, esse desafio é o resultado da hesitação causada pelo receio de resultados imprevisíveis da RP e da PP . Para outros, o desafio decorre de suas crenças sobre o que é matemática, sobre o ensino de matemática e o que significa saber matemática. Crenças sobre o que é propor e resolver problemas ou experiências pessoais enquanto alunos resolvendo problemas ou também experiências profissionais como professores que estão implementando a RP e a PP , também podem ter influência. Independentemente da origem, os professores em formação inicial e os professores experientes precisam de ajuda para desenvolver e sustentar suas práticas de RP e PP , e uma fonte dessa ajuda pode vir das comunidades de desenvolvimento profissional. Para a implementação de uma nova perspectiva os professores precisam estar equipados com as crenças correspondentes sobre o ensino e a aprendizagem de matemática. O objetivo desta palestra é apresentar alguns resultados de experiências de implementação de RP e PP , na formação inicial de professores através das disciplinas de prática de ensino, valendo-se de uma estrutura organizacional denominada de Aliança Professor- Pesquisador para a Investigação da Aprendizagem Matemática (APPIAM). Apresento a noção de cadeia de implementação de RP e PP (Koichu, Cooper, Widder, 2022) como uma sequência dinâmica de atividades pretendidas, planejadas, executadas e experimentadas, desenvolvidas e refletidas em conjunto, por pesquisadores em Educação Matemática (EM), professores em formação inicial, professores atuantes na Educação Básica (EB) e seus alunos, onde a natureza da atividade e seus objetivos podem mudar ao longo do tempo. A cadeia de implementação serve como um quadro analítico para investigar a implementação de recursos de RP e PP . A equipe de designers, que envolve pesquisadores em EM, ouvindo os professores e futuros professores desenvolve recursos de RP e PP destinados a alcançar alunos da EB por meio de seus professores. A atividade de RP e PP evolui ao longo da cadeia de implementação e então identificamos oportunidades de aprendizado mútuo que emergem de tensões das perspectivas sobre RP e PP apreendidos pelas diferentes partes envolvidas.

O Teorema Fundamental do Cálculo a partir da exploração de uma situação de cálculo de velocidades instantâneas

Guillermo Enrique Ramírez Montes
Universidade da Costa Rica, San Pedro, Costa Rica

Abstract

TBA

Mechanics

Dinâmica dos fluidos computacional utilizando OpenFOAM: um curso prático

Ciro Fraga Alegretti Cepetro e Rafael Gabler Gontijo
Unicamp e UnB, Campinas e Brasília, Brasil

Abstract

O uso de simuladores computacionais para obtenção das caraterísticas físicas de escoamentos é uma tendência que veio para ficar. Devido a não-linearidade intrínseca às equações de Navier-Stokes, muitos problemas de interesse científico, tecnológico e industrial no campo da dinâmica dos fluidos não são passíveis de tratamento analítico exigindo abordagens experimentais ou computacionais. A abordagem experimental, apesar de lidar com o fenômeno (escoamento) em seu estado puro, envolve custos com a montagem de protótipos e aquisição de sensores (instrumentação) que podem ser bastante elevados. Além disso, a confecção de uma bancada experimental para o estudo das características de um escoamento leva tempo e possui incertezas de medição associadas à possíveis interferências entre os sensores de medição e o fluido.

Mapeamento conforme para a solução de problemas de fronteira livre em equações diferenciais parciais

Roberto Ribeiro Santos Jr.
UFPR, Curitiba, Brasil

Abstract

O tratamento numérico de equações diferenciais parciais com fronteira livre é uma tarefa desafiadora. Este minicurso se propõe a apresentar um mapeamento conforme, reconhecido por sua eficiência no desenvolvimento de esquemas numéricos para resolver equações da hidrodinâmica. Embora amplamente conhecido na comunidade de ondas aquáticas, este mapeamento é pouco utilizado por estudantes e pesquisadores de outras áreas. Nosso objetivo neste minicurso é apresentar esta técnica de forma acessível para aqueles envolvidos com matemática aplicada e áreas correlatas. O minicurso abordará o mapeamento conforme para o tratamento numérico das equações de Euler não lineares, apresentando a dedução analítica das fórmulas e equações relevantes, assim como sua implementação computacional prática. Este. Este curso terá um foco prático com os códigos sendo implementados na linguagem Matlab.

Uma Introdução às Redes Neurais Informadas pela Física

Vinicius de Carvalho Rispoli
UnB, Brasília, Brasil

Abstract

A inteligência artificial baseada em aprendizado de máquina faz parte das nossas vidas cotidianas de várias formas, como por exemplo: carros autônomos e semi-autônomos, reconhecimento facial e de fala, auxílio a diagnósticos médicos, pesquisas do Google, recomendações de produtos e conteúdo nas redes sociais. Dentro do contexto científico essas mesmas ferramentas podem ser utilizadas para modelagem de problemas complexos relacionados a leis físicas. Em particular, as redes neurais informadas pela física (do inglês Physics-Informed Neural Networks - PINNs) são redes neurais que codificam equações modelo, como equações diferenciais parciais, como um componente da própria rede neural. As PINNs são hoje utilizadas para resolver equações diferenciais ordinárias, equações diferenciais parciais, equações diferenciais fracionárias, equações integrais e equações diferenciais estocásticas.

Number Theory

On Functional Graphs Over Finite Fields

Abílio Lemos
Federal Univertsity of Viçosa, Viçosa, Brazil
Joint Work With JOSIMAR J.R. AGUIRRE AND VICTOR G.L. NEUMANN

Abstract

The dynamic of iterations of polynomials over finite fields have attracted much attention in recent years in part due to their applications in cryptography and integer factorization methods like Pollard rho algorithm. We define the functional graph $f : \mathbb{F}_{q^2} \to \mathbb{F}_{q^2}$ as the directed graph $\mathcal{G}(f) = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \mathbb{F}_{q^2}$ and $\mathcal{E} = \{\langle x, f(x)\rangle \mid x \in \mathbb{F}_{q^2}\}$. We define the iterations of f as $f^{(0)}(x) = x$ and $f^{(n+1)}(x) = f\left(f^{(n)}(x)\right)$. Since f is defined over finite field, fixing $\alpha \in \mathbb{F}_q$, there are integers $0 \leq i < j$, minimal, such that $f^{(i)}(\alpha) = f^{(j)}(\alpha)$. In the case when $i > 0$, we call the list $\alpha, f(\alpha), f^{(2)}(\alpha), \ldots, f^{(i-1)}(\alpha)$ the pre-cycle and $f^{(i)}(\alpha), f^{(i+1)}(\alpha), \ldots, f^{(j-1)}(\alpha)$ the cycle of length $(j-i)$ or the $(j-i)$ cycle. If α is an element of a cycle, we call it a periodic element and, if $f(\alpha) = \alpha$ we say it is a fixed point. In this talk, we present some results in this topic and some results of the functional graph $\mathcal{G}(f)$ of the map $a \longmapsto f(a)$, where $f(X) = X\left(X^{q-1} - c\right)^{q+1}$, for $c \in \mathbb{F}_q^*$.

On Some Monomial Evaluation Codes And Their Duals

Cícero Carvalho
Federal Univertsity of Uberlândia, Uberlândia, Brazil
Joint Work With Hiram López And Gretchen Matthews (Virginia Tech)

Abstract

In this talk we would like to present some results on the parameters of a family of evaluation codes defined over the points of the so-called extended norm-trace curve, and their duals. We will also present some facts from Gröbner basis theory which, together with the concept of indicator functions, will be useful in studying this new class of codes.

Improvements To Warning's Theorem

David Leep
University Of Kentucky, Lexington, USA
Joint Work With Rachel Petrik

Abstract

Let $\mathbb{F}_q$ be the finite field with q elements, where q is a power of the characteristic p, and let $f = \{f_1, \ldots, f_r\}$ be a set of polynomials in $\mathbb{F}_q[x_1, \ldots, x_n]$ of degrees $d_1, \ldots, d_r$, respectively. Let $N(f, \mathbb{F}_q)$ denote the number of common zeros of $f_1, \ldots, f_r$ defined over $\mathbb{F}_q$.

The theorems due to Chevalley and Warning state that if $N(f, \mathbb{F}_q) > 0$ and if $n > d_1 + \cdots + d_r$, then $N(f, \mathbb{F}_q)$ is divisible by p and $N(f, \mathbb{F}_q) \geq q^{n-(d_1+\cdots+d_r)}$.

There are examples where $N(f, \mathbb{F}_q) = q^{n-(d_1+\cdots+d_r)}$ under the stated hypotheses, showing that this bound is best possible.

Heath-Brown proved that if $N(f, \mathbb{F}_q) = q^{n-(d_1+\cdots+d_r)}$, then the set of common zeros of f over $\mathbb{F}_q$ form an affine linear space (a subspace or a translate of a subspace).

Heath-Brown studied systems f where the set of common zeros of f over $\mathbb{F}_q$ does not form an affine linear space. In these cases, then subject to the above hypotheses, he showed that $N(f, \mathbb{F}_q) > q^{n-(d_1+\cdots+d_r)}$ and he gave some lower bound estimates for $N(f, \mathbb{F}_q)$, which improve the classical estimate of $q^{n-(d_1+\cdots+d_r)}$.

In this talk I will present theorems that improve the estimates of Heath-Brown.

Pairs Of Quadratic Forms Over P-Adic Fields

David Leep
University Of Kentucky, Lexington, USA
Joint Work With Rachel Petrik

Abstract

Let K be a p-adic field and let $Q_1, Q_2 \in K[x_1, \ldots, x_n]$ be quadratic forms in n variables with coefficients in K.

Heath-Brown proved that if $n = 8, Q_1, Q_2$ have a nontrivial common zero defined over K, and the variety defined by $\{Q_1, Q_2\}$ is nonsingular, then there exist $a, b \in K$, not both zero, such that $aQ_1 + bQ_2$ splits off at least 3 hyperbolic planes.

This rather technical theorem, whose proof is very long, was an important ingredient to Heath-Brown's proof that nonsingular pairs of quadratic forms in 8 variables defined over a number field satisfy the Hasse Principle. More concretely: Suppose that $\{Q_1, Q_2\}$ is a pair of quadratic forms in 8 variables defined over a number field F and assume that the variety defined by $\{Q_1, Q_2\}$ is nonsingular. If Q_1, Q_2 have a nontrivial zero defined over each nonarchimedean completion of F and also over each real completion of F (if any), then Q_1, Q_2 have a nontrivial zero defined over F.

Heath-Brown's theorem on pairs of quadratic forms over p-adic fields raised many questions in my mind, including why his proof was so difficult. Also, it seemed that this result for $n = 8$ should be part of a bigger theorem for general values of n.

This talk will explore the situation for general values of n.

Additive Forms Over Totally Ramified Extensions Of Q2

Drew Dnucan
John Carroll University, University Heights, USA

Abstract

Let $\Gamma^*(d,k)$ be the least number s so that any additive form $a_1x_1^d + \ldots + a_sx_s^d$ over the field k has a nontrivial zero. We would like to know that $\Gamma^*(d,k) \leq d^2+1$ holds for every p-adic field k. Thus far, progress on this has been restricted to degrees of ramification at most 2 . In this talk we will see that, in the case of $d = 2m, m$ odd, and k a totally ramified extension of $\mathbb{Q}_2$ of arbitrarily high degree, with relatively little work we can obtain this bound, and even improve on it.

TBA

Fabio Brochero
UFMG, Belo Horizonte, Brazil

Abstract

TBA

The Corner Element Of Generalized Numerical Semigroups

Guilherme Tizziotti
Federal University of Uberlândia, Uberlândia, Brasil
JOINT WORK WITH M. BERNARDINI AND W. TENÓRIO.

Abstract

In this talk we present the concept of corner element of a generalized numerical semigroup (GNS), which extends in a sense the idea of conductor of a numerical semigroup to GNS. Also, we present properties of this new notion and its relations with existing invariants in the literature, we provide an algorithm to compute all the GNSs with fixed corner, and lower and upper bounds on the number of GNSs having a fixed corner element.

N THE EXCEPTIONAL SET OF TRANSCENDENTAL ENTIRE FUNCTIONS IN SEVERAL VARIABLES

Jean Carlos de Aguiar Lelis
UFPA, Belém, Brasil
JOINT WORK WITH ALVES, MARQUES, AND TROJOVISKY

Abstract

In this work, among other things, we prove that any subset of $\overline{\mathbb{Q}}^m$ under complex conjugation and which contains the origin) is the exceptional set of un- countable many transcendental entire functions over $\mathbb{C}^m$with rational coefficients. This result solves a several variables version of a question posed by Mahler for transcendental entire functions.

THE DENSITY OF SPECIAL ELEMENTS IN FINITE FIELDS, ON AVERAGE

Lucas Reis
Universidade, Belo Horizonte, Brasil

Abstract

In this talk we discuss the behaviour of some arithmetic functions related to the density of special elements in finite field extensions (normal, k-normal and primitive elements). Namely, if q is a fixed prime power and $f(n)$ denotes the proportion of elements in the finite field $\mathbb{F}_{q^n}$ with one of those specified properties, we discuss the behaviour of the arithmetic function f(n), on average. In all cases, the function $f(n)$ has a positive mean value, in the sense that the limit

$$\lim_{x|+\infty} \frac{1}{x} \sum_{n \leq x} f(n),$$

exists and it is positive. We provide further information on this mean value and propose some possible questions for future research. The techniques are simple and rely on basic rudiments of the theory of arithmetic functions. **Keywords:** normal elements, mean values, arithmetic functions.

N THE SEEDS AND THE GREAT-GRANDCHILDREN OF A NUMERICAL SEMIGROUP

Maria Bras-Amorós
UNIVERSITAT ROVIRA I VIRGILI TARRAGONA, Catalunha, Spain

Abstract

We present a revisit of the seeds algorithm to explore the semigroup tree. First, an equivalent definition of seed is presented, which seems easier to manage. Second, we determine the seeds of semigroups with at most three left elements. And third, we find the great-grandchildren of any numerical semigroup in terms of its seeds. The algorithm has been used to prove that there are no Eliahou semigroups of genus 66, hence proving the Wilf conjecture for genus up to 66. We also found three Eliahou semigroups of genus 67. One of these semigroups is neither of Eliahou-Fromentin type, nor of Delgado's type. However, it is a member of a new family suggested by Shalom Eliahou.

INFINITE DESCENDANT NODES IN A NUMERICAL SEMIGROUP TREE

Mariana Rosas
URV, Catalunha, Spain

Abstract

Motivated by studying the growth of the sequence n_g that counts the number of numerical semigroups of genus g, in this talk we will see how to organize such semigroups in a tree so that at each level g of the tree there are numerical semigroups of genus g. In this tree - which seems to only grow but the proof of this is an open problem - we will characterize the nodes with infinite descendants, which appear to be rare but support the tree.

FACTORIZATION THEORY VIA ADDITIVE COMBINATORICS

Sávio Ribas
UFOP, Ouro Preto, Brasil

Abstract

We will introduce the relationship between factorization theory (focused on non-uniqueness) and zero-sum problems in additive combinatorics. Often the elements of a ring (such as the ring of integers of a number field) or of a monoid H can be written in several different ways as a finite product of irreducibles. The *set of lengths* of $a \in H$, $L(a) = \{k \in \mathbb{N} \,|\, \exists\, u_1, \cdots, u_k$ irreducible s.t. $a = u_1 \cdots u_k\}$, and the system of *set of lengths*, $\mathcal{L}(H) = \{L(a) \,|\, a \in H\}$, are means of describing the non-uniqueness of factorization in H. It is conjectured that $\mathcal{L}(\mathcal{B}(G))$, where $\mathcal{B}(G)$ is the monoid of zero-sum sequences over an abelian group G, completely determines G except for a few small cases. Furthermore, for $k \in \mathbb{N}$, let $\mathcal{U}_k(H)$ be the set of all $m \in \mathbb{N}$ such that there are irreducible $u_1, \cdots, u_k, v_1, \cdots, v_m$ satisfying $u_1 \cdots u_k = v_1 \cdots v_m$. Define $\lambda_k(H) = min\mathcal{U}_k(H), \rho_k(H) = sup\mathcal{U}_k(H)$ (k-th elasticity of H) and $\rho(H) = sup\,\dfrac{\rho_k(H)}{k}$ (elasticity of H). It is known that $\lambda_k(H)$ can be written in terms of $\rho_k(H)$, which in turn can be lower and upper bounded by the Davenport constant of G, which is defined as the maximum length of a minimal zero-sum sequence. The sets $\mathcal{U}_k(H)$ are usually well structured (they look like arithmetic progressions). For example, $\mathcal{U}_k(H) = [\lambda_k(H), \rho_k(H)]$ if and only if the ideal class group G of H is finite abelian. This makes the k-th elasticity one of the most important invariants to describe the non-uniqueness of the factorization in H. We will present the main results and conjectures about $\rho_k(H)$.

INTEGRAL VALUES OF GENERATING FUNCTIONS OF RECURRENCE SEQUENCES

Victor Gonzalo Lopes Neumann
UFU, Uberlândia, Brasil
JOINT WORK WITH MICHAEL KNAPP AND ABÍLIO LEMOS

Abstract

Suppose that $a_0, a_1, \cdots$ is an integer sequence which satisfies a recurrence relation with constant coefficients, and let $T(x) = f(x)/g(x)$ be its generating function, where $f(x)$ and $g(x)$ have no common factors in $\mathbb{Z}[x]$. In this talk, we study the problem of finding the rational values of x such that $T(x)$ is an integer. We say that such a number is *good* for the sequence. Our first main result is that if $g(x)$ has at least two different irreducible factors, or if $g(x)$ has a single irreducible factor of degree at least 3, then the sequence has only finitely many good values. We also study sequences of the form $0, 1, \cdots$ for which the recurrence relation has order 2. Among other results, we show that under a mild condition on the recurrence relation, the sequence has infinitely many good values, and we give a constructive method to find all of them.

Probability

Posters

Cohomology and Partial Differential Equations

Artur Jorge Marinho
UFG, Goiânia, Brasil

Abstract

TBA

Necessary Conditions for Trudinger-Moser Inequality on Complete Riemannian Manifolds

Júlio Cesar Pereira França
UFG, Goiânia, Brasil

Abstract

TBA

Weakly singular problem in nonreflexive fractional Orlicz-Sobolev space

Luana de Carvalho Maciel
UFG, Goiânia, Brasil

Abstract

TBA

Non transitive self-similar metabelian groups

Melissa de Sousa Luiz
Unicamp, Campinas, Brasil

Abstract

TBA

Fluxo Redutor de Curvas

Miriam Cristina Ferreira Furtado
UFG, Goiânia, Brasil

Abstract

TBA

On elliptic Kirchhoff-Boussinesq type problems with exponential growth

Romulo Diaz Carlos
University of Brasilia, Brazil
Joint work with Giovany M. Figuereido (UnB)

Abstract

In this presentation we are concerned with existence of nontrivial solutions for the following classes of problems

$$\Delta^2 u \pm \Delta_p u = f(u) \text{ in } \Omega, \quad \text{and } u = \Delta u = 0 \text{ on } \partial\Omega,$$

where $\Omega \subset \mathbb{R}^4$ is a smooth bounded domain, $2 < p < 4$ and $f : \mathbb{R} \to \mathbb{R}$ is a superlinear continuous class function with exponential subcritical or critical growth. We apply the Nehari manifold method in order to prove the main result.

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