Seminário de Análise

Extremal functions for Trudinger–Moser inequalities

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Abstract.

In this paper we prove existence of extremal functions for certain Trudinger–Moser inequalities on the whole plane. Precisely, by using blow-up analysis, we show that the supremum

$$\sup_{u\in W^{1,2}(\mathbb{R}^2):\,\|u\|_{1,2}=1}\int_{\mathbb{R}^2}\left[e^{4\pi(1+\alpha\|u\|_2^2)u^2}-1\right]\mathrm{d}x \qquad (*)$$

is attained for all $\alpha\in[0,1)$, where $W^{1,2}(\mathbb{R}^2)$ is the standard first order Sobolev space endowed with the Sobolev norm $\|u\|_{1,2}^2=\|\nabla u\|_2^2+\|u\|_2^2$ and $\|u\|_2$ is the usual L^2- Lebesgue norm. This kind of inequality was originally proposed by Adimurthi and O. Druet [1] in bounded domains of $\mathbb{R}^2$ and extended by Y. Yang [4] to high dimensional case. This current paper improved and complemented the results of [2, 3], where the authors have proved that the supremum in $(*)$ is finite.

References

[1] Adimurthi and O. Druet, *Blow-up analysis in dimension 2 and a sharp form of Trudinger-Moser inequality*, Comm. Partial Differential Equations **29** (2004), 295–322.

[2] M. de Souza and J. M. do Ó, *A sharp Trudinger–Moser type inequality in* $\mathbb{R}^2$, Trans. Amer. Math. Soc. **366** (2014), 4513–4549.

[3] M. de Souza and J. M. do Ó, *Trudinger–Moser inequality on the whole plane and extremal functions*, Commun. Contemp. Math. **18** (2016), 1550054, 32 pp.

[4] Y. Yang, *A sharp form of Moser-Trudinger inequality in high dimension*, J. Funct. Anal. **239** (2006), 100–126.