DINAMICAL SYSTEMS SEMINAR

An introduction to uniform structures

André Caldas
University of Brasilia

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Abstract. A *metric space* is a set equipped with a notion of distance: a *metric*. With this metric, we can talk about *convergence*, *continuity* and also, *uniform continuity*, for instance. We talk about the ball $B_\varepsilon(x)$, centered at x and with radius $\epsilon > 0$. Changing the center, we can form new balls, also with radius ε. Given $x_n \to x$ and $y_n \to y$, we can ask ourselves about which one converges “faster”. When we say that a function is *uniformly continuous*, intuitively we are saying that when x_n approaches x and y_n approaches y, both with the “same velocity”, then $f(x_n)$ and $f(y_n)$ also approach $f(x)$ and $f(y)$, both with the “same velocity”.

A *topological space* is a set with a *topology*, that allow us to generalize the concepts of *convergence* and *continuity*. But just the topology does not allow one to talk about *uniform continuity*, for example. Let V_x and V_y be neighborhoods of x and y. Without and additional structure, there is no arguing about which one, V_x or V_y, is “bigger then the other”. Without more information besides the topology, there is no way to compare the speed that x_n converges to x with the speed that y_n converges to y. Is there a way to “transport” a neighborhood V_x of x in such a way to transform it, preserving its “size”, into neighborhoods of other points? This is roughly the idea behind *uniform structures*. It is a natural generalization of metric spaces that allow us to speak, for example, of uniform convergence.

This is an introductory talk that can be understood by any student that already knows what *uniform continuity* is, and that has some notion of what a topology is: *open sets* and *neighborhoods of a point*.

References

[1] John L. Kelley. *General Topology*. Graduate Texts in Mathematics **(27)**. Springer-Verlag. New York, 1975.