Seminário de Álgebra

Indecomposable, free and separating $O(n)$-invariants of several matrices

Artem Lopatin
UNICAMP

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Abstract. All vector spaces are over an infinite field $\mathbb{F}$ of the characteristic $p = \operatorname{char}\mathbb{F} \neq 2$. To define the algebra of $O(n)$-invariants of matrices, consider the polynomial algebra

$$R = R_n = \mathbb{F}[x_{ij}(k) \,|\, 1 \le i, j \le n,\ 1 \le k \le d]$$

together with $n \times n$ *generic* matrices

$$X_k = \begin{pmatrix} x_{11}(k) & \cdots & x_{1n}(k) \\ \vdots & & \vdots \\ x_{n1}(k) & \cdots & x_{nn}(k) \end{pmatrix}.$$

Denote by $\sigma_t(A)$ the t^{th} coefficient of the characteristic polynomial of A. As an example, $\operatorname{tr}(A) = \sigma_1(A)$ and $\det(A) = \sigma_n(A)$. The action of the orthogonal group

$$O(n) = \{A \in M_n \,|\, AA^T = I_n\}$$

over R is defined by the formula: $g \cdot x_{ij}(k) = (g^{-1}X_k g)_{ij}$, where $(A)_{ij}$ stands for the $(i,j)^{\text{th}}$ entry of an matrix A, M_n is the space of all $n \times n$ matrices over $\mathbb{F}$. The set of all elements of R that are stable with the respect of the given action is called the algebra of *$O(n)$-invariants of matrices* and is denoted by $R^{O(n)}$. This algebra is generated by $\sigma_t(b)$, where $1 \le t \le n$ and b runs over all monomials in the generic matrices $X_1, \ldots, X_d$ and the transposed generic matrices.

Note that in case $\mathbb{F} = \mathbb{R}, \mathbb{C}$ the $O(n)$-orbits of $M(n)^{\oplus d}$ are separated by elements of $R^{O(n)}$. In the case of an arbitrary characteristic $p \neq 2$ the ideal of relations between generators of $R^{O(n)}$ was described in [3, 4]. Sets of generators of orthogonal invariants of matrices and some generalization of them were considered in [1, 2, 5] in case $n \le 4$. Denote by $D_{\max}(R^{O(n)})$ the maximal degree of *indecomposable* invariants, i.e., the maximal degree of elements of a minimal generating set for $R^{O(n)}$. We obtained that

- the algebra $R^{O(n)}$ is not a polynomial for all $d > 1$;
- $D_{\max}(R^{O(n)}) \to \infty$ when $d \to \infty$, in case $2 < \operatorname{char}\mathbb{F} \le n$;
- a minimal separating set in case of skew-symmetric 3×3 matrices is described (together with Ronaldo Ferreira).

References

[1] A.A. Lopatin, *Orthogonal invariants of skew-symmetric matrices*, Linear and Multilinear Algebra **59** (2011), 851–862.

[2] A.A. Lopatin, *On minimal generating systems for matrix $O(3)$-invariants*, Linear and Multilinear Algebra **59**, (2011) 87–99.

[3] A.A. Lopatin, *Relations between $O(n)$-invariants of several matrices*, Algebras and Representation Theory **15** (2012), 855–882.

[4] A.A. Lopatin, *Free relations for matrix invariants in the modular case*, Journal of Pure and Applied Algebra **216** (2012), 427–437.

[5] A.A. Lopatin, *Minimal system of generators for $O(4)$-invariants of two skew-symmetric matrices*, Linear and Multilinear Algebra, **66** (2018), no. 2, 347–356.