Seminário de Álgebra

Homological VS and n-$(n+1)$-$(n+2)$ Conjectures

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23/11/18
Sexta-feira
14:30 Horas

Auditório do MAT

Abstract. A group G is of type FP_n, with $n \geq 0$, if there is a projective resolution

$$\mathcal{P} : P_n \to \ldots \to P_i \to P_{i-1} \to \ldots \to P_0 \to \mathbb{Z} \to 0$$

of trivial $\mathbb{Z}G$-module $\mathbb{Z}$ such that each $\mathbb{Z}G$-projective module P_i is finitely generated for $0 \leq i \leq n$. The property FP_n is a homological version of another homotopic property called F_n. A group G is of homotopic type F_n if, and only if, G is of homological type FP_n and finitely presented.

There is a conjecture called n-$(n+1)$-$(n+2)$ Conjecture, that we call Homotopic n-$(n+1)$-$(n+2)$ Conjecture in this work. This claims that, for $n \geq 0$, given two short exact sequences of groups $N_1 \hookrightarrow G_1 \overset{\pi_1}{\twoheadrightarrow} Q$ and $N_2 \hookrightarrow G_2 \overset{\pi_2}{\twoheadrightarrow} Q$, if N_1 is of homotopic type F_n, both G_1 and G_2 are of homotopic type F_{n+1} and Q is of homotopic type F_{n+2}, then the fiber product P of π_1 and π_2 is of homotopic type F_{n+1}.

Related to the latter conjecture there is another one called Virtual Surjection Conjecture, that we also call Homotopic in this work. This claims that, for $n \geq 2$, given $G_1, \ldots, G_k$ groups of homotopical type F_n, where $n \leq k$, and $P \subseteq G_1 \times \ldots \times G_k$ a subgroup that virtually surjects on every n factors, i.e. for every $1 \leq i_1 < \ldots < i_n \leq k$ the image of P under canonical projection $P \to G_{i_1} \times \ldots \times G_{i_k}$ has finite index, then P is of type F_n.

These conjectures are unsolved until now, but Benno Kuckuck proved some interesting related results in 2012.

Motivated by Kuckuck's work, we have proposed the **Homological** n-$(n+1)$-$(n+2)$ Conjecture and **Homological** Virtual Surjection Conjecture that the assertions are the same of the conjectures above replacing F_n with FP_n. We have proved analogous results to Kuckuck's results, but using spectral sequences in some of then. Furthermore the work here is quite different from

Kuckuck's work because our groups are not finitely presented in general.

In special we have proved Homological 1-2-3 Conjecture when Q is finitely presented and Homological Virtual Surjection Conjecture when $n = 2$ (Homological VSP Criterion).

This is a work join with Dessislava H. Kochloukova.

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