Research

DYNAMICAL SYSTEMS SEMINAR

Second homotopy group and invariant geometry of flag manifolds

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Abstract. We will begin this talk by briefly presenting some of my favourite tools in algebraic topology: homotopy groups, the exact sequence of homotopy groups and the Hopf fibration.

We will then use these tools to explicitly compute generators of the second homotopy group of my favourite manifolds: flag manifolds of a compact Lie group. This result generalizes and clarifies a result in [2]. It shows how "rubber-band" topology can, in the presence of symmetry, single out very rigid objects.

If time allows, we will finish by exploring the rigidity of these generators of the second homotopy group. More precisely, we will characterize when the generators in the same homotopy class have the same geometry for all invariant metrics. This is done by exploring the action of Weyl group of the flag manifold, generalizing a result of [3]. This shows that some aspects of "continuum" invariant geometry can, in the presence of symmetry, be reduced to the study of discrete objects.

In this exposition, we will mostly stick with classical flag manifolds (type A) and, if time allows, we will present at the end an example with a quaternionic flag manifold (type C). We remark that the topology singling out very rigid objects and the study of a continuum object being reduced to discrete objects are characteristic of situations with a lot of symmetry and, thus, are recurring themes in Lie theory.

This is joint work with Lino Grama, IMECC-UNICAMP.

References

[1] L. Grama and L. Seco: Second homotopy and invariant geometry of flag manifolds. Submitted in 2018. arXiv:1803.01290

[2] F. E. Burstall and J. H. Rawnsley: *Twistor Theory for Riemannian Symmetric Spaces*, Springer Lect. Notes in Math.1424 (1990).

[3] M. Patrão, L.A.B. San Martin The isotropy representation of a real flag manifold: Split real forms, Indag. Math. **26**, 547-579 2015.