Graduate

SEMINARIO DE ÁLGEBRA

On identities of associative Lie nilpotent algebras on three generators

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Abstract. Let Φ be an associative and commutative unital ring, $\mathcal{A}$ the free associative unital algebra of countable rank over Φ, and $\mathcal{T}^{(n)}$ the T-ideal of $\mathcal{A}$ generated by the commutator $\Big[\big[\ldots[a_1,a_2],\ldots,a_{n-1}\big],a_n\Big]$, where $[a,b]=ab-ba$ and $n\geqslant 2$. The conditions for the inclusion of the product $\mathcal{T}^{(m)}\mathcal{T}^{(k)}$ into $\mathcal{T}^{(n)}$ $(n<m+k)$ and related questions were studied in papers by Latyshev (1965), Volichenko (1978), Sharma and Srivastava (1990), Bapat and Jordan (2013), Grishin and Pchelintsev (2015), Deryabina and Krasilnikov (2017, 2018). Briefly, it is known that $3\mathcal{T}^{(m)}\mathcal{T}^{(k)}\subseteq\mathcal{T}^{(m+k-1)}$ whenever at least one of the integers m,k is odd. On the other hand, $\mathcal{T}^{(2m)}\mathcal{T}^{(2k)}\nsubseteq\mathcal{T}^{(2m+2k-1)}$. Let $\mathcal{A}_3$ be the free 3-generated unital subalgebra of $\mathcal{A}$ and $\mathcal{T}^{(n)}\cap\mathcal{A}_3=\mathcal{T}_3^{(n)}$. Dangovski [1] conjectured that $\mathcal{T}_3^{(m)}\mathcal{T}_3^{(k)}\subseteq\mathcal{T}_3^{(m+k-1)}$ holds for all m,k over a field of characteristic 0. Independently, Pchelintsev [2] proved this inclusion under the assumption $\frac{1}{6}\in\Phi$. We prove that the named inclusion holds with no restrictions on the ground ring Φ. In consequence, we establish that the relatively free Lie nilpotent algebra $\mathcal{A}_3/\mathcal{T}_3^{(n)}=\mathcal{A}_3^{(n)}$ satisfies the family of multilinear identities of the form $\mathrm{c}_{k_1}x_1\mathrm{c}_{k_2}x_2\ldots\mathrm{c}_{k_{t-1}}x_{t-1}\mathrm{c}_{k_t}=0$, where the sequence $k_1,\ldots,k_t$ runs the set of all partitions $n-1=k_1+\cdots+k_t$ $(k_i\in\mathbb{N})$ and each $\mathrm{c}_k=\mathrm{c}_k(y_1,\ldots,y_k\mid z_1,\ldots,z_k)$ stands for the commutator $\mathrm{c}_k=\Big[\Big[\ldots\big[[y_1,z_1]\,y_2,z_2\big]\,y_3,\ldots,z_{k-1}\Big]\,y_k,z_k\Big]$. In particular, $\mathcal{A}_3^{(n)}$ is strong Lie nilpotent of index n and the T-ideal $\mathcal{T}_3^{(n-1)}$ lies in the center $Z\big(\mathcal{A}_3^{(n)}\big)$. Moreover, the additive module of $\mathcal{A}_3^{(n)}$ has no torsion.

References

[1] R. R. Dangovski, On the maximal containments of lower central series ideals, Preprint (2016), arXiv:1509.08030v2.

[2] S. Pchelintsev, Relatively free associative algebras of ranks 2 and 3 with Lie nilpotency identity and systems of generators of some T-spaces, Preprint in Russian with English summary (2018), arXiv:1801.07771.