Graduate

Probability Seminar

A New Class of Solutions for the Generalized Langevin Equation

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Abstract. We extend the work of [3], [2], [4], [5] and present a new class of solutions to the Generalized Langevin Equation (ELG): the Generalized Orstein-Uhlenbeck Process of the Floating Exponential Type (GOU-FE), for which two of the solutions analized in [5] are special cases. We propose a resolvent function and prove that it yields a solution for the ELG when the driving Lévy noise is a jump diffusion with finite second moment, [1] an α-stable process with stability parameter $1 < \alpha \leq 2$,[2] or, in general, any Lévy process when the stochastic integration is defined in terms of convergence in probability,[4] by presenting a memory function for which the proposed resolvent function is solution for a Voltera integro-differential equation. We also prove a recurrence relation that yields alternatives for estimating and simulating the OUG-FE through corollaries that deal with the α-stable, jump diffusion and jump diffusion with normal jumps cases. Particularly, we also prove that when $O(h^3) \to 0$, where h is the discretization step of the recurrence relation, the GOU-FE has a discrete version given by a stationary ARMA$(2, 1)$ process that can be used to model, simulate and estimate stationary processes for which the finite sample autocorrelation function presents a damped oscillation, what is not possible with the processes analyzed in [5], which either present an exponentially decreasing autocorrelation or do not present stationarity in their discrete versions.

References

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[5] Stein, J., Lopes, S. R. C., & Medino, A. V. 2016. Continuous Processes Derived from the Solution of Generalized Langevin Equation: Theoretical Properties and Estimation. *Journal of Statistical Computation and Simulation*.