Undergraduate

DYNAMICAL SYSTEMS SEMINAR

Topology Using Nets

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Abstract. In the beginning, there were *sequences*…
…then, Cantor and Tietze said:

Let there be "open sets".

And *General Topology* was made.

With sequences, a point x was said to be in the (sequential) closure of a set A when one is able to "reach" x using a convergent sequence $x_n \in A$. A set is (sequentially) compact when every sequence has a convergent subsequence. A function f is (sequentially) continuous at x when for every sequence $x_n \to x$, we have that $f(x_n) \to f(x)$, and so on. Everything was very dynamic back then! One could always hide a sequence and go hopping from x_n to x_{n+1} and so on… But it turned out that sequences had some annoying disadvantages.

Open sets are not so "dynamic". They are in fact, quite "static". The function is continuous when the inverse image of an open set is open. The point x is in the closure of a set A when every open set containing x intersects A.

Wikipedia told me that Moore and Smith introduced the concept of *nets* in 1922. I guess they wanted to think about topology in that "dynamic fashion" sequences allowed them to do in the old days. In many cases, one takes a definition or proof done with sequences, substitutes everything with nets, and you get a working definition or proof for topological spaces where sequences fail.

The point is in the closure when there is a net converging to it. The function is continuous when for every net $x_\lambda \to x$, $f(x_\lambda) \to f(x)$. A set is compact when every net has a convergent subnet…

In this talk, I shall introduce the concepts of nets. Talk about its advantages, and also talk about some caveats. What can I do, and how? What CAN'T I do?

References

[1] G.K. Pedersen. *Analisys Now*. Graduate Texts in Mathematics 118. Springer, 2001.