Research

Probability Seminar

Large deviations principles for Lévy processes in the small noise limit.

Andre de Oliveira Gomes
Department of Mathematics - UNICAMP

Date: 25/10/2019

Time: 14:15 Hours

Mini-Auditorium MAT

Abstract. The weak convergence approach to Large Deviations Theory can be traced to earlier works of Fleming on Stochastic Control where the derivation of variational formulas for functionals of Markov processes played a fundamental role within the interplay of the dynamic programming principle. Later on Dupuis, Ellis, Budhiraja and collaborators developed a robust ensemble of sufficient criteria for the large deviations regime built on variational formulas for functionals of Brownian Motions and Poisson random measures. Their constructions were successfully used to derive large/moderate deviations principles in a diverse spectrum of applications. We present the basic toolkit of this theory in order to analyze the large deviations regime of a multi-scale stochastic system of differential equations driven by a Lévy process in the small noise limit. Multi-scale stochastic equations are popular tools at the level of design of mathematical models for complex systems where it is appropriate to consider a scale separation of variables with low/high degrees of freedom. Typical examples are two-scale systems of coupled stochastic differential equations in Climatology where the fast variables are variables related with the weather short-time window forecast and the slow variables are climatic proxy-data that can be used to trace climatic transitions from different meta-stable states. We derive the large deviations principle for the slow variable of the stochastic system with jumps that is under consideration written in terms of a rate function that reflects the underlying averaging principle of the system. We underline that a large deviations principle is a much stronger statement than a strong averaging principle. The construction of the rate function actually will boil down to the verification of a Khasminkii's type of averaging principle for shifted versions by controlled processes of the multi-scale system due to the weak convergence arguments that we use in the derivation of the LDP. Our ultimate goal is to solve the Kramers problem for the slow variable, which means to characterize the law and the expectation of the first time the slow variable process exits the domain of attraction of the stable state of the underlying dynamical system. Our analysis relies on the construction of the potential height related to this problem and on the sharp analysis of the upper/lower bounds for the law of the first exit time random variable in terms of the potential. This talk is based on the works [1] and [2].

References

[1] A. O. Gomes. M. Högele. The Kramers problem for SDEs driven by small, accelerated Lévy noise with exponentially light jumps. *https://arxiv.org/abs/1904.02125*

[2] P. Catuogno. A. O. Gomes. Moderate averaged deviations for a multi-scale system with jumps and memory. *https://arxiv.org/abs/1909.10894*