Dynamical Systems Seminar

Compact Riemannian Manifold with Prescribed Isometry Group

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Abstract. The problem to find a metric with some prescribed geometric information are the most interesting topic in Differential Geometry. That kind of question is a particular case of the study of the total landscape of the set of Riemannian structures on a given compact manifold. For example, on any compact manifold of any dimension there exist Riemannian metrics of constant scalar curvature and they form an infinite dimensional family for $n \geq 3$. There is no standard tools to deal with that class of problems. Although, variational methods, PDE, fixed point theorems serve as starting point.

In the present talk we going to deal with the problem to find a metric on a compact manifold with prescribed isometry group. We will show a theorem due to Jean Pierre Bourguignon that says that if G is a compact Lie subgroup of the group of diffeomorphism of the manifold then there exist a metric g such that the isometry grupo is G. We would like to point out that the proof relies on the group action theoretical methods. Motivated by this result we are currently investigating the related problem to find a metric on homogeneous spaces with some more geometric or algebraic information prescribed. The prototype cases are compact Lie groups equipped with biinvariant metric or more generally symmetric spaces.

References

[1] J P. Bourguignon; Une Stratification de L'Espace Des Structures Riemanniennes, Compositio Mathematica, tome 30, no 1 (1975), p. 1–41

[2] D. G. Ebin; On the Space of Riemannian Metrics, Bull. Amer. Math. Soc. V. 74, N. 5 (1968), 1001–1003

[3] S. Helgason, Differential Geometry, Lie Groups, and Symmetric Spaces, American Mathematical Society; First Edition edition (June 12, 2001)