On Finiteness of some verbal subgroups in profinite groups

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Abstract

A group word $w = w(x_1, x_2 \ldots, x_n)$ is an element of the free group F of rank n. If G is any group, we denote by G_w the set of all elements of G obtained by replacing $x_1, x_2, \ldots, x_n$ in w by arbitrary elements $g_1, g_2, \ldots, g_n$ of G, and by $w(G)$ the subgroup generated by G_w. The word w is said to be concise in a class of groups $\mathcal{C}$ if, for each $G \in \mathcal{C}$ such that G_w is finite, then $w(G)$ is also finite.

In the context of profinite groups G, we write $w(G)$ to denote the *closed* subgroup generated by all w-values in G. In [1], E. Detomi, B. Klopsch and P. Shumyatsky introduced a stronger version of the classical notion of conciseness. We say that a group word w is *strongly concise* in a class of profinite groups $\mathcal{C}$ if, for every $G \in \mathcal{C}$ such that the cardinality of G_w is less than $2^{\aleph_0}$, it follows that $w(G)$ is finite.

In this talk, we consider profinite groups admitting a word w such that $|G_w| < 2^{\aleph_0}$ and $w(G)$ is generated by finitely many w-values. We prove that Engel words of the form $[v, {}_n u]$ are strongly concise, for some u and v in F. We cover the cases where u and v are chosen suitably between commutator-closed words and powers of lower central and weakly rational words.

This is a joint work with Pavel Shumyatsky.

References

[1] E. Detomi, B. Klopsch, P. Shumyatsky, Strong conciseness in profinite groups, J. Lond. Math. Soc. (2). 102 (2020) 977–993.

[2] J. P. P. Azevedo, P. Shumyatsky, On finiteness of some verbal subgroups in profinite groups, J. Algebra, to appear.

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