Analysis Seminar

Regularity estimates for fully nonlinear doubly degenerate elliptic models

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Abstract. We establish $C^{1,\beta}_{\mathrm{loc}}$ regularity estimates for bounded solutions of a class of fully nonlinear elliptic PDEs with non-homogeneous degeneracy, whose toy model is given by

$$[|Du|^p + \mathfrak{a}(x)|Du|^q]\,\mathcal{M}^+_{\lambda,\Lambda}(D^2u) = f(x,u) \quad \text{in} \quad \Omega,$$

for a bounded domain $\Omega \subset \mathbb{R}^N$, and suitable data $p, q \in (0, \infty)$, $\mathfrak{a}$ and f. Such estimates simplify and generalize, to some extent, earlier ones via a different strategy (cf. [1], [5] and [6]). Our approach is based on geometric tangential methods and makes use of a refined oscillation mechanism combined with compactness and scaling techniques. Finally, we present some connections of our findings with a variety of nonlinear free boundary problems in the theory of elliptic PDEs (see [2] and [4]). This Lecture is based on joint work with G.C. Ricarte (UFC-Brazil), see [3].

References

[1] D.J. Araújo, G.C. Ricarte, E.V. Teixeira, *Geometric gradient estimates for solutions to degenerate elliptic equations.* Calc. Var. Partial Differential Equations 53 (2015), 605-625.

[2] J.V. da Silva, R.A. Leitão Júnior and G.C. Ricarte, *Geometric regularity estimates for fully nonlinear elliptic equations with free boundaries.* Mathematische Nachrichten, Vol. 294(1) 2021 p. 38-55 DOI: 10.1002/mana.201800555.

[3] J.V. da Silva and G.C. Ricarte, *Geometric gradient estimates for fully nonlinear models with non-homogeneous degeneracy and applications.* Calc. Var. Partial Differential Equations 59, 161 (2020).

[4] J.V. da Silva and H. Vivas, *The obstacle problem for a class of degenerate fully nonlinear operators*, To appear in Revista Matemática Iberoamericana, 2021 DOI 10.4171/rmi/1256.

[5] C. De Filippis, *Regularity for solutions of fully nonlinear elliptic equations with nonhomogeneous degeneracy.* Proceedings of the Royal Society of Edinburgh: Section A Mathematics, 1-23. DOI:10.1017/prm.2020.5.

[6] C. Imbert and L. Silvestre, *$C^{1,\alpha}$ regularity of solutions of some degenerate fully non-linear elliptic equations.* Adv. Math. 233 (2013), 196–206.