Research

Analysis Seminar

Nonlinear boundary problem for Harmonic functions in higher dimensional Euclidean half-spaces

Marcelo Fernandes de Almeida
Universidade Federal de Sergipe

Date: April 9th, 2021

Time: 10:00 am

On-line at Zoom: https://bit.ly/3f0cwxu

Abstract. In this talk we are interested on solvability of the problem

$$\begin{cases} -\Delta u = 0 & \text{in } \mathbb{R}^n_+ \\ \dfrac{\partial u}{\partial \nu} = V(x')u + b|u|^{\rho-1}u + f & \text{on } \partial\mathbb{R}^n_+ \end{cases}$$

with high singular data f and potential V on boundary $\partial\mathbb{R}^n_+$ of half-space $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n \,|\, x_n > 0\}$ for $n > 2$. More precisely, inspired at [3] and [2] we introduce a new functional space based in weak-Morrey spaces and we shown solvability of the problem when data $f \in$ weak-$\mathcal{M}_p^{(n-1)(\rho-1)/\rho}(\mathbb{R}^{n-1})$ and potential $V \in$ week-$\mathcal{M}_\ell^{n-1}(\mathbb{R}^{n-1})$ are small for all $(n-1)/(n-2) < \rho < \infty$. Our result recover the supercritical range $n/(n-2) \leq \rho < \infty$ and external force f and potential V taken over the boundary are new, in view of strictly inclusions $L^\lambda \subsetneq L^{\lambda,\infty} \subsetneq \mathcal{M}_p^\lambda \subsetneq$ week-$\mathcal{M}_p^\lambda$ for $1 < p < \lambda < \infty$. Also, we shown symmetries of solutions and from Campanato's lemma we conclude that $u \in C^{0,\alpha}(\overline{\mathbb{R}^n_+})$ is Hölder continuous, provided that $f \in \mathcal{M}_p^{(n-1)(\rho-1)/\rho}(\mathbb{R}^{n-1})$ and $V \in \mathcal{M}_\ell^{n-1}(\mathbb{R}^{n-1})$ are taken in Morrey spaces.

References

[1] M. F. de Almeida, L. S. M. Lima: Adams' trace principle on Morrey-Lorentz spaces over β-Hausdorff dimensional surfaces. Preprint (2019).

[2] P. Quittner and W. Reichel, Very weak solutions to elliptic equations with nonlinear Neumann boundary conditions, Calc. Var. Partial Differential Equations **32** (2008), no. 4, 429–452.

[3] M. F. de Almeida, L. C. F. Ferreira and J. C. Precioso, On the heat equation with nonlinearity and singular anisotropic potential on the boundary, Potential Anal. **46** (2017), no. 3, 589–608.