Analysis Seminar

Adjoint semigroups for abstract delay equations and local nonlinear analysis

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On-line at Zoom

Abstract.

Given a $\mathcal{C}_0$-semigroup T_0 on a non-reflexive Banach space X, perturbation theory for adjoint semigroups provides a canonical embedding of X into a larger Banach space $X^{\odot\star}$ that depends on both X as well as T_0. In the theory of delay equations, this embedding has been applied successfully to obtain a useful correspondence between, on the one hand, certain classes of delay equations and, on the other hand, continuous linear or nonlinear perturbations into $X^{\odot\star}$ of certain translation semigroups T_0 on X. Most of this work was done under the assumption of sun-reflexivity of X with respect to T_0. As far as necessary, I will recall the precise meaning of this assumption.

For the purpose of this talk, an *abstract* delay equation is a delay equation with the property that the histories $x_t : [-h, 0] \to Y$ at times $t \geq 0$ take values in a Banach space Y that need not be finite dimensional. In this case, the assumption of sun-reflexivity may be violated. Examples of this situation were studied in [2] for the class of abstract renewal equations, and in [3] for a class of abstract delay differential equations. In turn, these studies motivated the abstract approach taken in [4], where the notion of an admissible perturbation into $X^{\odot\star}$ for the semigroup T_0 on X plays a central role. I will explain how this notion relates to the subspace $X^{\odot\times}$ of $X^{\odot\star}$ already introduced in [5]. I will also explain how admissibility can be used as a weaker substitute for the assumption of sun-reflexivity.

As an illustration, I will compare the construction of a center manifold of an equilibrium in the non-sun-reflexive case with known results [1] for the sun-reflexive case. A systematic use of the space $X^{\odot\times}$ facilitates their generalization with relatively little effort. I give sufficient conditions for the existence of appropriate spectral decompositions of X and $X^{\odot\times}$ without assuming that the linearized semiflow is eventually compact. A center manifold theorem for the class of abstract delay differential equations then follows as a special case.

In conclusion, I briefly comment on work, existing and in progress, on some motivating example problems, and on a number of ideas for future directions.

References

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[2] O. Diekmann and M. Gyllenberg (2008). Abstract delay equations inspired by population dynamics. In: Functional Analysis and Evolution Equations. Birkhäuser, Basel.

[3] S. G. Janssens (2019). A class of abstract delay differential equations in the light of suns and stars. Part I. Preprint. `https://arxiv.org/abs/1901.11526`

[4] S. G. Janssens (2020). A class of abstract delay differential equations in the light of suns and stars. Part II. Preprint. `https://arxiv.org/abs/2003.13341`

[5] J. M. A. M. van Neerven (1992). The Adjoint of a Semigroup of Linear Operators. Lecture Notes in Mathematics 1529. Springer-Verlag, Berlin.