Self-similar abelian groups.**

Tulio Santos
Instituto Federal Goiano

Abstract

A self-similar group is a group G acting on a one-rooted m-regular tree $\mathcal{T}_m$ in such a way that the states of its elements are themselves elements of G. The permutation group induced on the first level of the tree has $s \geq 1$ orbits; when $s = 1$ the group is said to be transitive. Famous examples of transitive self-similar groups include the infinite torsion 2-group of Grigorchuk and the Gupta-Sidki p-groups.

Nekrashevych and Sidki characterized all transitive self-similar free abelian subgroups of finite rank of the group of automorphisms of the binary tree $\mathcal{T}_2$, [3]. Later Brunner and Sidki [2] conducted the most complete study of transitive abelian self-similar groups showing for example that the closure of such groups under the full diagonal operations, $\alpha \mapsto (\alpha, \alpha, \cdots, \alpha)$, of the group of automorphisms of $\mathcal{T}_m$ is again abelian. This lead to an important translation of transitive self-similar abelian groups to modules of the m-adic algebra $\mathbb{Z}_m[[x]]$. The generalization to the intransitive case, where the group has $s > 1$ orbits, requires a careful study of a free monoid Δ of rank s, of partial diagonal operations acting on the group of automorphisms of $\mathcal{T}_m$. We show that in this setting, if A is a self-similar abelian group in $\mathcal{A}_m$ then the centralizer of $\Delta(A)$ in $\mathcal{A}_m$ is additively a finitely generated $\mathbb{Z}_n[[\Delta]]$ module and derive as consequence, the torsion subgroup of A is also a self-similar group of finite exponent, a divisor of n. Furthermore, we extend recent constructions of self-similar free abelian groups of infinite enumerable rank to examples of such groups which are also Δ-invariant.

This is a work in progress.

Joint work with Said Sidki and Alex Dantas

References

[1] A. C. Dantas; T M. G. Santos; S. N. Sidki, Intransitive self-similar groups, *Journal of Algebra* 567, 2021, 564-581.

[2] A. M. Brunner; S. N. Sidki, Abelian state-closed subgroups of automorphisms of m-ary trees, *Groups, Geometry, and Dynamics* 4, 2010, 455-471.

[3] V. Nekrashevych; S. N. Sidki, Automorphisms of the binary tree: state-closed subgroups and dynamics of 1/2-endomorphisms, *Groups: topological, combinatorial and aritmetic aspects, London Mathematical Society Lecture Note Series* 311. Cambridge University Press, 2004, 375-404.