Research

Dynamical Systems Seminar

Quotients of the holomorphic 2-ball and the turnover

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Date: 02/09/2021
Time: 15:00

Abstract. We say that a 4-manifold (4-orbifold) has complex hyperbolic structure if it is uniformized by the complex hyperbolic plane $\mathbb{H}^2_{\mathbb{C}}$. This space is a Klein geometry that can be modeled as the unit ball in $\mathbb{C}^2$ endowed with its group of biholomorphisms, which coincide with $\mathrm{PU}(2,1)$.

The complex variant of the Gromov-Lawson-Thurston (GLT) conjecture states that an oriented disc bundle over a closed oriented surface admits a complex hyperbolic structure if, and only if, the inequality $|e/\chi| \leq 1$ holds, where e is the Euler number of the bundle and χ is the Euler characteristic of the surface. Both directions of the conjecture are still open.

We construct non-rigid disc orbibundles with complex hyperbolic structures over 2-orbifolds. These are the first non-rigid examples of this kind. More precisely, in the character variety of representations of the surface group of the 2-orbifold in $\mathrm{PU}(2,1)$, these disc orbibundles form a 2-dimensional region. Passing to the surface level (using Selberg's Lemma) we obtain examples supporting the complex GLT conjecture, including examples of complex hyperbolic structures on trivial and cotangent bundles of surfaces. This is joint work with Carlos Grossi.

References

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[Bot] Hugo C. Botós, *Orbifolds and orbibundles in complex hyperbolic geometry.* arXiv:2011.09372

[GKL] W. M. Goldman, M. Kapovich, B. Leeb, Complex hyperbolic manifolds homotopy equivalent to a Riemann surface, Comm. Anal. Geom. 9, no. 1 (2001), 61–95.