PI-Superalgebras with Superinvolution: Hook and Strip Theorems

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Abstract

Let $A = A_0 \oplus A_1$ be a superalgebra over a field F of characteristic zero. A superinvolution in A is a graded linear application $\# : A \to A$ such that $(c^{\#})^{\#} = c$ for all $c \in A$ and $(ab)^{\#} = (-1)^{deg(a)deg(b)} b^{\#} a^{\#}$ for all homogeneous elements $a, b \in A_0 \cup A_1$, where $deg(d)$ is the homogeneous degree of $d \in A_0 \cup A_1$. In this case, we say that A is a $\#$-superalgebra.

The study of superalgebras with some superinvolution and their identities is of the great interest for several areas of Mathematics.

One of the important results of the theory of polynomial identities is the celebrated Hook Theorem, which was proven by Amitsur and Regev in [1]. There exist also versions of this theorem for the case of $\mathbb{Z}_2$-graded identities and identities with involution that were proved by Regev and Giambruno in [2].

In 1979, Regev showed in [3] that, if A is an algebra that satisfies the Capelli identity of rank k, then the sequence of cocharacters of A has bounded height by $k-1$. In the PI-theory, this result is known as The Strip Theorem.

These results are related with the using of theory of group representations for the understanding of a behaviour of identities and, moreover, they have various applications in PI-theory and in other areas of Mathematics.

The main goal of this talk is to present a version of the Hook and Strip Theorems for the case of superidentities with superinvolution .

For a superalgebra with superinvolution over a field F of characteristic zero, the ideal of superidentities with superinvolution is completely defined by multilinear identities that have a structure of $S_{\langle n\rangle}$-modulo, where $\langle n\rangle = (n_1, n_2, n_3, n_4)$, $n = n_1 + n_2 + n_3 + n_4$, and each n_i corresponds to the quantity of homogeneous $\#$-symmetric or $\#$-antisymmetric variables. The behavior of these $\#$-superidentities may be described by the corresponding cocharacter

$$\chi_{\langle n\rangle}(A) = \sum_{\langle\lambda\rangle \vdash \langle n\rangle} m_{\langle\lambda\rangle}\chi_{\langle\lambda\rangle},$$

where $\langle\lambda\rangle = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ is a multipartition of $\langle n\rangle$ and $\lambda_i \vdash n_i$ is a partition of n_i.

Theorem A: *(Hook Theorem for $\#$-superalgebras)* Let A be a $\#$-superalgebra. If A is a PI-algebra (i.e., A also satisfies some non-trivial ordinary identity), then there exist integers $d_i, l_i \geq 0$, with $i \in \{1, 2, 3, 4\}$, such that the n-th cocharacter, $\chi_{\langle n\rangle}(A)$, is contained in a quadruple hook

$$H_4(n) = (H(d_1, l_1), H(d_2, l_2), H(d_3, l_3), H(d_4, l_4)),$$

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that is,

$$\chi_{\langle n\rangle}(A) = \sum_{\substack{\langle\lambda\rangle\vdash n \\ \langle\lambda\rangle\in H_4(n)}} m_{\langle\lambda\rangle}\chi_{\langle\lambda\rangle},$$

where $\langle\lambda\rangle \in H_4(n)$ means $\lambda_i \in H(d_i, l_i)$, for all $i \in \{1,2,3,4\}$, that is, $\lambda_{d_{i+1}} \leq l_i$.

Theorem B: *(The Strip Theorem for #-superalgebras)* Let A be a #-superalgebra and consider $\chi_{\langle n\rangle}(A) = \sum_{\langle\lambda\rangle\vdash\langle n\rangle} m_{\langle\lambda\rangle}\chi_{\langle\lambda\rangle}$ its n-th cocharacter. If A is a PI-algebra, then A satisfies $Cap_{\langle k\rangle}$, $k = k_1 + k_2 + k_3 + k_4$, the Capelli #-identity of rank $\langle k\rangle$, if, and only if, $m_{\langle\lambda\rangle} = 0$ whenever $h(\langle\lambda\rangle) \geq \langle k\rangle$

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Referências

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