Research

Algebra Seminar

On Finite Groups Admitting Coprime Automorphisms

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Acesso à sala virtual:

Abstract. Let G be a finite group admitting an automorphism ϕ. Denote by G_ϕ the centralizer of ϕ in G and by $G_{-\phi}$ the set $\{x^{-1}x^\phi \mid x \in G\}$. The subgroup generated by $G_{-\phi}$ will be denoted by $[G, \phi]$. There are many results relating the structure of the group G and the properties of G_ϕ and $G_{-\phi}$.

In this talk, we present results bounding the exponent of G and $[G, \phi]$. They are concentrated in finite groups that admit a coprime automorphism, with special attention to odd order groups that admit an involutory automorphism.

More specifically, if G is a finite group of odd order admitting an involutory automorphism ϕ, the following results were obtained: suppose that G_ϕ is nilpotent of class c. If $x^e = 1$ for each $x \in G_{-\phi}$ and the subgroup $\langle x, y\rangle$ has derived length at most d for every $x, y \in G_{-\phi}$, then the exponent of $[G, \phi]$ is bounded in terms of c, d and e. On the other hand, if G_ϕ has rank r and $x^e = 1$ for each $x \in G_{-\phi}$, then the exponent of $[G, \phi]$ is bounded in terms of e and r.

Furthermore, assume that G is a finite group admitting a coprime automorphism ϕ of order n. We prove that, if every element from $G_\phi \cup G_{-\phi}$ is contained in a ϕ-invariant subgroup of exponent dividing e, then the exponent of G is bounded in terms of e and n. To demonstrate this result, Lie-theoretic tools created by Zelmanov (see [4]) were used. In addition, we extend the first result as follows: suppose that G_ϕ is nilpotent of class c. If $x^e = 1$ for each $x \in G_{-\phi}$ and any two elements of $G_{-\phi}$ are contained in a ϕ-invariant soluble subgroup of derived length d, then the exponent of $[G, \phi]$ is bounded in terms of c, d, e and n.

Finally, we also extend the second result to the case in which the order of the automorphism may be an arbitrary prime. We prove that, if G is a finite group admitting a coprime automorphism ϕ of prime order p such that G_ϕ has rank r and $x^e = 1$ for each $x \in G_{-\phi}$, then the exponent of $[G, \phi]$ is bounded in terms of e, p and r.

This is a joint work with Pavel Shumyatsky. It is important to point out that the results mentioned above in this talk are published in [1], [2] and [3].

References

[1] S. R. S. Rodrigues, P. Shumyatsky, Exponent of a finite group of odd order with an involutory automorphism, Archiv der Mathematik, **113** (2019), 113-118.

[2] S. R. S. Rodrigues, P. Shumyatsky, Exponent of a finite group admitting a coprime automorphism, J. Pure Appl. Algebra, **224** (2020), 106370.

[3] S. R. S. Rodrigues, P. Shumyatsky, Exponent of a finite group admitting a coprime automorphism of prime order, J. Group Theory, **24** (2021), 635-642.

[4] P. Shumyatsky, Applications of Lie ring methods to group theory, Nonassociative Algebra and Its Applications, (Eds R. Costa et al.), Marcel Dekker, New York, (2000), 373-395.