SEMINÁRIO DE TEORIA DOS NÚMEROS

Quarta-feira, 15 de setembro

Horário Especial: 10 horas

SOME RECENT RESULTS ON THE DIOPHANTINE EQUATIONS WITH POWER SUMS

GÖKHAN SOYDAN

Bursa Uludag University (Turquia)

The Bernoulli numbers are among the most interesting and important number sequences in mathematics. They first appeared in the posthumous work “Ars Conjectandi" (1713) by Jakob Bernoulli (1654-1705) in connection with sums of powers of consecutive integers. Bernoulli numbers are particularly important in number theory, especially in connection with Fermat's last theorem. They also appear in the calculus of finite diferences (Nörlund (1924)), in combinatorics, and in other fields. Let us define $S_k(x) = 1^k + 2^k + \dots + x^k$ where x and k are nonnegative integers. Equations of the shape $S_k(x) = y^n$ have been considered by several authors (Bennett, Györy, Hajdu, Pintér, Schäffer, Tijdeman, ...). In 1985, J. W.S. Cassels solved the Diophantine equation $(x-1)^3 + x^3 + (x+1)^3 = y^2$ in integers x and y, showing that the only solutions satisfy x = 0; 1; 2 and 24. In this talk, we first give old/new results about the Diophantine equation $(x + 1)^k + (x + 2)^k + \dots + (x+r)^k = y^n$ (*). We prove that (*) has finitely many integer solutions with $x, y > 0$, $n > 1$, $k \neq 3$. Then, extending some old results about the Diophantine equation $(x - d)^2 + x^2 + (x + d)^2 = y^n$ (**), we give an explicit formula for all positive integer solutions of the equation (**) when n is na odd prime and $d = p^r$, $p > 3$ a prime. These works are joint with Daniele Bartoli and Maohua Le.