A SYSTEM OF LOCAL/NONLOCAL $p-$LAPLACIANS: THE EIGENVALUE PROBLEM AND ITS ASYMPTOTIC LIMIT AS $p \to \infty$

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Abstract. In this talk I am going to present some result obtained in collaboration with J.V. da Silva and L.H de Miranda. Given $p \in (1, \infty)$, we prove the existence and simplicity of the first eigenvalue λ_p and its corresponding eigenvector (u_p, v_p), for the following local/nonlocal PDE system

$$
\left\{
\begin{array}{rcll}
-\Delta_p u + (-\Delta)_p^r u & = & \frac{2\alpha}{\alpha+\beta}\lambda |u|^{\alpha-2}|v|^{\beta} u & \text{in} \quad \Omega \\
-\Delta_p v + (-\Delta)_p^s v & = & \frac{2\beta}{\alpha+\beta}\lambda |u|^{\alpha}|v|^{\beta-2} v & \text{in} \quad \Omega \\
u & = & 0 & \text{on} \quad \mathbb{R}^N \setminus \Omega \\
v & = & 0 & \text{on} \quad \mathbb{R}^N \setminus \Omega,
\end{array}
\right.
\tag{0.1}
$$

where $\Omega \subset I\!R^N$ is a bounded open domain, $0 < r, s < 1$ and $\alpha(p) + \beta(p) = p$. Moreover, we address the asymptotic limit as $p \to \infty$, proving the explicit geometric characterization of the corresponding first $\infty-$eigenvalue, namely λ_∞, and the uniformly convergence of the pair (u_p, v_p) to the $\infty-$eigenvector (u_∞, v_∞). Finally, the triple $(u_\infty, v_\infty, \lambda_\infty)$ verifies, in the viscosity sense, a limiting PDE system.