Undergraduate

Number Theory Seminar

Analytic Continuations, Riemann's Zeta Function and the Prime Number Theorem - Part 1/4

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Time: XXhYYm

This lecture will be held online on Zoom:
https://us02web.zoom.us/j/XYZW...

Abstract. This is a series of four lectures aiming to present a complete proof of the Prime Number Theorem for undergraduate students. Opposed to the Atiyah-Singer index theorem or Fermat's last theorem, the Prime Number Theorem is one of the few landmark mathematical results whose its proof is fully accessible at the undergraduate level. It states that $\pi(x) \equiv$ the number of primes less than x, behaves asymptotically like $x/\log x$, more precisely,

$$\lim_{x\to\infty} \frac{\pi(x)}{x/\log x} = 1.$$

Our presentation is based on the techniques introduced by Bernhard Riemann on its epoch-making 8-pages paper "On the Number of Primes Less Than a Given Magnitude", which allow us to construct an unexpected and powerful connection between discrete and continuous mathematical structures. Almost no knowledge on Number Theory is required to understand completely the arguments that will be presented during these lectures. Moreover, in order to make the presentation as accessible as possible, at the beginning, we will briefly remind/introduce the needed background material on Complex Analysis such as: Weierstrass M-test; locally uniform convergence; general infinite products; Euler's formula; analytic continuation; and the Riemann zeta function. Next, following the reference [1], we explain the steps of the modern approach to prove the Prime Number Theorem. This involves talking about Chebyshev's function ψ and its integrated cousin ψ_1, the von Mangoldt function, the Gamma function, Jacobi Theta function and Poisson Summation Formula. At the end, we will show that the proof of the Prime Number Theorem, actually, boils down to the asymptotic analysis of: the two Chebyshev's functions, near to the infinity; and Riemann zeta function on the boundary of the critical strip.

References

[1] E. Stein and R. Shakarchi. *Complex Analysis*, first edition, Princeton University Press, (2003).

[2] H. M. Edwards. *Riemann's Zeta Function*, Academic Press, New York, Dover Publications, Inc., Mineola, NY, (2001).