## ANALYSIS SEMINAR

## Elliptic systems involving Schrödinger operators with vanishing potentials

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We prove the existence of a bounded positive solution of the following elliptic system involving Schrödinger operators

$$\begin{cases} -\Delta u + V_1(x)u = \lambda \rho_1(x)(u+1)^r (v+1)^p & \text{in } \mathbb{R}^N \\ -\Delta v + V_2(x)v = \mu \rho_2(x)(u+1)^q (v+1)^s & \text{in } \mathbb{R}^N, \\ u(x), v(x) \to 0 & \text{as } |x| \to \infty \end{cases}$$

where  $p, q, r, s \ge 0$ ,  $V_i$  is a nonnegative vanishing potential, and  $\rho_i$  has the property (H) introduced by Brezis and Kamin [1]. As in that celebrated work we will prove that for every R > 0 there is a solution  $(u_R, v_R)$  defined on the ball of radius R centered at the origin. Then, we will show that this sequence of solutions tends to a bounded solution of the previous system when R tends to infinity. Furthermore, by imposing some restrictions on the powers p, q, r, s without additional hypotheses of integrability on the weights  $\rho_i$ , we obtain a second solution using variational methods. In this context we consider two particular cases: a gradient system and a Hamiltonian system. This work is motivated in part by [2] and [3].

Joint work with Denilson Pereira and Juan Arratia.

## References

- [1] H. Brezis and S. Kamin. Sublinear elliptic equations in  $\mathbb{R}^N$ . Manuscripta Math. 74 (1992), 87-106.
- [2] J. A. Cardoso, P. Cerda, D. S. Pereira and P. Ubilla. Some Brezis-Kamin type problems and applications to Schrödinger Equation involving vanishing potentials. Discrete Contin. Dyn. Syst, 2021, 41(6): 2947-2969.
- [3] M. Montenegro. The construction of principal spectral curves for Lane-Emden systems and applications. Ann. Sc. Norm. Super. Pisa Cl. Sci. 4 série, tome 29 (2000), 193-229.