

**THE RIBES-ZALESSKII PROPERTY FOR GRAPHS OF FREE  
GROUPS WITH CYCLIC EDGES.**

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**Abstract:**

Let  $G$  be a group and let  $n$  be a natural number. The group  $G$  is said to have the Ribes-Zaleskii property  $RZ_n$  if for arbitrary finitely generated subgroups  $H_1, \dots, H_n$  of  $G$ , the product  $H_1 \cdots H_n$  is closed in the profinite topology on  $G$ .

Thus property  $RZ_1$  means that  $G$  is subgroup separable (LERF) and property  $RZ_2$  means that  $G$  is double coset separable. If  $G$  satisfies  $RZ_n$  for all natural numbers  $n$  then  $G$  is called product separable. In 1993 Ribes and Zaleskii proved that free groups are product separable. This confirmed a conjecture of Pin and Reutenauer, that was motivated by questions from Semigroup Theory. In my talk I will discuss recent joint work with Lawk Mineh, where we investigated product separability for groups hyperbolic relative to families of product separable subgroups. In particular we show that limit groups, finitely generated Kleinian groups and balanced fundamental groups of finite graphs of free groups with cyclic edge groups are all product separable.