Block Theory for Profinite Groups

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Abstract

If $G = \varprojlim G/N$ is a profinite group and k is a field, the representation theory of G seeks to understand the (pseudocompact) modules for the (pseudocompact) algebra $k[[G]] = \varprojlim k[G/N]$. As with finite dimensional algebras, we may write $k[[G]]$ as a direct product of indecomposable algebras, known as the *blocks* of G. Rather than worry about the $k[[G]]$-modules all at once, it is frequently convenient to study the representation theory of these blocks one at a time, as certain blocks may have a much easier theory. The difficulty of a block B of G is measured by a pro-p subgroup D of G, unique up to conjugation in G, called the *defect group* of B: morally, the smaller D is, the easier B is (extreme case: B is a simple algebra if, and only if, $D = 1$). A fundamental theorem in the block theory of finite groups (part of the so-called "local/global principal") is the Brauer Correspondence, which states that the blocks of the finite group G with a given defect group D are in natural correspondence with the blocks of the subgroup $\mathrm{N}_G(D)$ with defect group D. I will explain a bit about block theory of finite and profinite groups, and extend the Brauer Correspondence to blocks of arbitrary profinite groups.

Joint work with Ricardo Franquiz (UFMG)