Centralizers of intransitive self-similar abelian groups.**

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Abstract

A self-similar group is a group G acting on a one-rooted m-regular tree $\mathcal{T}_m$ in such a way that the states of its elements are themselves elements of G. Famous examples of self-similar groups include the infinite torsion 2-group of Grigorchuk and the Gupta-Sidki p-groups. A successful method for constructing self-similar groups is based on the notion of virtual endomorphisms of groups, introduced by Nekrashevych and Sidki [4]; however this produced groups which act transitively on the first level of the tree. We extended the notion of a single virtual endomorphism to a set of virtual endomorphisms by which all self-similar groups could be constructed.

Nekrashevych and Sidki characterized all self-similar free abelian subgroups of finite rank of the group of automorphisms of $\mathcal{T}_2$ [4]. Later Brunner and Sidki [2] conducted the most complete study of transitive abelian self-similar groups showing for example that the closure of such groups under the full diagonal operations, $\alpha \mapsto (\alpha, \alpha, \cdots, \alpha)$, of the group of automorphisms of $\mathcal{T}_m$ is again abelian. This lead to an important translation of transitive self-similar abelian groups to modules of the m-adic algebra $\mathbb{Z}_m[[x]]$. The generalization to the intransitive case requires a careful study of a free monoid Δ of partial diagonal operations acting on the group of automorphisms of $\mathcal{T}_m$. We show that in this setting, the closure of a self-similar abelian group A under Δ continues to be self-similar abelian.

In [3] Berlatto and Sidki showed that the centralizer of an abelian recurrent group is its own topological closure, in particular is abelian. We have studied the centralizer structure of an intransitive abelian self-similar group and its closure by Δ and described recursively the centralizer of an intransitive cyclic state-closed subgroup of $Aut(\mathcal{T}_4)$. The analysis is based on the orbit-type of the group. In particular we show that the centralizer of the double addition machine is not abelian, but the centralizer of its closure by Δ is abelian.

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