# On the Free Jordan Algebras

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**Abstract**

Let $K$ be a field of characteristic zero. For integers $n$, $D \geq 1$, let $J_n(D)$ be the degree $n$ component of the free Jordan algebra $J(D)$ over $D$ generators. A conjecture for the character (in particular for the dimension) of the $GL(D)$-module $J_n(D)$ is proposed.

Let $\mathfrak{sl}_2 J(D)$ be the Tits-Allison-Gao construction of $J(D)$ (this is a certain version of the famous Tits-Kantor-Koecher construction of $J(D)$), see [1]. Two natural conjectures for the homology of Lie algebra $\mathfrak{sl}_2 J(D)$ are stated, and each of them implies the previous conjecture.

We also provide some numeric evidence that the proposed formulae for dimensions of $J_n(D)$ reflects the known phenomenons: for $D = 3$ and $n = 8$ the conjecture predicts that the space of special identities has dimension 3, which is correct: those are the famous Glennie's Identities [2]. Similarly for $D = 4$ the conjecture agrees that some tetrads are missing in $J(4)$, as it has been observed by Cohn [3].

# References

[1] B.N. Allison and Y. Gao, *Central quotients and coverings of Steinberg unitary Lie algebras,* Canad. J. Math. 48 (1996) 449-482.

[2] C.M. Glennie, *Some identities valid in special Jordan algebras but not valid in all Jordan algebras,* Pacific J. Math. 16 (1966) 47-59.

[3] P. M. Cohn, *On a generalization of the Euclidean algorithm,* Proc. Cambridge Philos. Soc. 57 (1961) 18-30.