A topological toolbox for Sobolev maps

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Abstract

Classical works by F. Bethuel and by F. Hang and F-H. Lin have identified the local and global topological obstructions that prevent smooth maps from being dense in the Sobolev space $W^{1,p}(M^m; N^n)$ between two Riemannian manifolds when p < m. They are related to the extension of continuous maps from subsets of M^m to N^n . In this talk I will present some work in progress with P. Bousquet (Toulouse) and J. Van Schaftingen (UCLouvain), inspired from the notions of modulus introduced by B. Fuglede and degree for VMO maps by H. Brezis and L. Nirenberg. I shall explain how one can decide whether a specific Sobolev map $u: M^m \to N^n$ can be approximated or not by smooth ones, even in the presence of topological obstructions from M^m or N^n .

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