The Nirenberg problem for the disk

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Abstract

The problem of prescribing the Gaussian curvature on compact surfaces is a classic one, and dates back to the works of Berger, Moser, Kazdan and Warner, etc. The case of the sphere receives the name of *Nirenberg problem* and has deserved a lot of attention in the literature. In the first part of the talk we will review the known results about compactness and existence of solutions to that problem.

If the domain is the disk $\mathbb{D}$, a boundary condition is in order. The most natural one is to prescribe also the geodesic curvature $h(x)$ of the boundary. This problem reduces to solve a semilinear elliptic PDE under a nonlinear Neumann boundary condition.

First we perform a blow-up analysis of the solutions. We will show that, if a sequence of solutions blow-up, it tends to concentrate around a unique point $p \in \partial\mathbb{D}$. We are able to give conditions on such point that, quite interestingly, involve the derivatives of $K(x)$ but depend on $h(x)$ in a nonlocal way. This is joint work with A. Jevnikar, R. López-Soriano and M. Medina.

Secondly, we will give existence results. We will show how the blow-up analysis developed before can be used to compute the Leray-Schauder degree of the problem.

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