Uniqueness and positivity issues in a subhomogeneous indefinite elliptic problem

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Abstract

We deal with uniqueness and positivity properties for the problem

$$(P_\lambda) \qquad \begin{cases} -\Delta_p u = \lambda u^{p-1} + a(x) u^{q-1} & \text{in} \quad \Omega, \\ u \geq 0 & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

where Ω is a bounded domain of $\mathbb{R}^N$, Δ_p is the p-Laplacian operator, $\lambda \in \mathbb{R}$, $a \in C(\overline{\Omega})$ changes sign and $1 < q < p$ (the so-called *subhomogeneous* case). These conditions on a and q enable the existence of *dead core* solutions and, consequently, multiplicity of nontrivial solutions may occur. We discuss uniqueness results in accordance with the sign of λ. Some extensions to more general right-hand sides will be considerded as well. Finally, the existence of positive solutions is analyzed with respect to a and q.

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