

Gradient estimates for fully nonlinear PDEs with non-homogeneous degeneracy

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Abstract

In this Lecture we present $C_{\text{loc}}^{1,\beta}$ regularity estimates for bounded solutions of a class of fully nonlinear elliptic equations with non-homogeneous degeneracy, whose simplest model case is given by $[|Du|^p + \mathbf{a}(x)|Du|^q] \Delta u = f(x, u)$ in Ω , for a bounded and open set $\Omega \subset \mathbb{R}^N$, and appropriate data $p, q \in (0, \infty)$, \mathbf{a} and f . Such regularity estimates simplify and generalize, to some extent, earlier ones via different modus operandi (cf. [1], [3] and [5]). Our approach is based on geometric tangential methods and makes use of a refined oscillation mechanism combined with compactness and scaling techniques. In the end, we also present some connections of our results with geometric free boundary problems (see, [4]) and relevant nonlinear models in the theory of elliptic PDEs (see, [2]).

References

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