Gradient behaviour for large solutions to semilinear elliptic problems

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Abstract

For any $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ smooth bounded domain, $p > 1$ and f Lipschitz function, it is well known that there exists a unique $u \in C^2(\Omega)$ that solves

$$\begin{cases} -\Delta u + |u|^{p-1}u = f & \text{in } \Omega, \\ u = +\infty & \text{on } \partial\Omega. \end{cases} \tag{1}$$

Moreover the first and the second order terms of the asymptotic expansion of u near the boundary are explicitly described, as well as the first order asymptotic expansion of the normal derivative of u.

The main result that we present provides not only the second order behaviour of the gradient of the large solution of (1), but also the complete asymptotic expansion of all the singular terms of u and ∇u, for every arbitrary smooth domain, every $p > 1$ and every $f \in W^{1,\infty}(\Omega)$. In particular we show that there exists an explicit corrector function S, finite sum of singular terms, such that

$$z := u - S \in W^{1,\infty}(\Omega).$$

Moreover we prove that

$$\forall\ \bar{x} \in \partial\Omega \qquad z(\bar{x}) = 0 \quad \text{and} \quad \lim_{\delta \to 0} \frac{z(\bar{x} - \delta\nu(\bar{x}))}{\delta} = 0,$$

where ν is the outward unit normal to $\partial\Omega$. The previous result is contained in the following paper.

References

[1] S. Buccheri, Gradient behaviour for large solutions to semilinear elliptic problems, *Annali di Matematica Pura ed Applicata* **198**, 1013-1040 (2019).

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