

# A Formalization of the $Z$ property in Coq

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## Abstract

Rewriting theory is a well established model of computation equivalent to the Turing machines, and the most well known rewriting system is the  $\lambda$ -calculus. Confluence is an important and undecidable property related to the determinism of the computational process. Direct proofs of confluence are, in general, difficult to be done. Therefore, alternative characterizations of confluence can circumvent this difficulty for different contexts. This is the case of the so called  $Z$  property, which has been successfully used to prove confluence in several situations such as the  $\lambda$ -calculus with  $\beta\eta$ -reduction, extensions of the  $\lambda$ -calculus with explicit substitutions, the  $\lambda\mu$ -calculus, etc. In this work we present a direct and constructive proof that the  $Z$  property implies confluence. In addition, we formalized our proof and an extension of the  $Z$  property, known as the Compositional  $Z$ , in the Coq proof assistant.

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