Classification of immersed Weingarten spheres in homogeneous three-manifolds

Jose Antonio Gálvez[*]
Departamento de Geometría y Topología
Universidad de Granada

Abstract

We study immersed spheres in a homogeneous three-manifold M whose mean, extrinsic and Gauss curvatures satisfy an elliptic Weingarten relation $W(H, K_e, K) = 0$. When the isometry group of M has dimension 4, we show that if the unique inextendible rotational surface S in M that satisfies this equation and touches its rotation axis orthogonally has bounded second fundamental form, then every immersed sphere satisfying $W(H, K_e, K) = 0$ must be rotational. This proves, among other results, that any elliptic Weingarten sphere immersed in $\mathbb{H}^2 \times \mathbb{R}$ is a rotational sphere and also, as a particular case, recovers the Abresch-Rosenberg classification of constant mean curvature spheres in M.

[*]e-mail: `jagalvez@ugr.es`