

Short-time existence for the network flow

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Abstract

This paper contains a new proof of the short-time existence for the flow by curvature of a network of curves in the plane. Appearing initially in metallurgy and as a model for the evolution of grain boundaries, this flow was later treated by Brakke [4] using varifold methods. There is good reason to treat this problem by a direct PDE approach, but doing so requires one to deal with the singular nature of the PDE at the vertices of the network. This was handled in cases of increasing generality by Bronsard-Reitich [5], Mantegazza-Novaga-Tortorelli [18] and eventually, in the most general case of irregular networks by Ilmanen-Neves-Schulze [11]. Although the present paper proves a result similar to the one in [11], the method here provides substantially more detailed information about how an irregular network “resolves” into a regular one. Either approach relies on the existence of self-similar expanding solutions found in [20]. As a precursor to and illustration of the main theorem, we also prove an unexpected regularity result for the mixed Cauchy-Dirichlet boundary problem for the linear heat equation on a manifold with boundary. This is joint work with Jorge Lira, Rafe Mazzeo and Alessandra Pluda.

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