

Several aspects of critical metrics for quadratic curvature functionals

Miguel Brozos Vázquez*
Department of Mathematics
Universidade da Coruña
Ferrol, Campus de Esteiro

Abstract

It is well known that, on a closed oriented manifold M , critical metrics for the total scalar curvature functional $g \mapsto \int_M \tau \, dvol_g$ are Einstein. Generalizing this functional, one can consider the scalar invariants τ^2 , $\|\rho\|^2$, $\|R\|^2$, $\Delta\tau$, that form a basis of the space of second order curvature invariants and give rise to the quadratic curvature functionals

$$\Phi_{a,b,c} : g \mapsto \int_M (a\tau^2 + b\|\rho\|^2 + c\|R\|^2) \, dvol_g, \text{ for } a, b, c \in \mathbb{R}.$$

In this talk we will first consider cone metrics and analyze the conditions for being critical to show examples of cone metrics that are critical for all quadratic curvature invariants. Afterwards, we will concentrate on 3-dimensional homogeneous manifolds and show that there is a critical metric for any $\Phi_{a,b,c}$. Special attention will be paid to functionals with zero energy, that are closely related to Ricci solitons in this context.

References

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*e-mail: miguel.brozoz.vazquez@udc.gal