The Allen-Cahn equation in the sphere: geometric and variational properties

Rayssa Caju*
Department of Mathematics
UFPB/ University of Chicago
João Pessoa, Brazil/ Chicago, USA

Abstract

The Allen-Cahn equation establishes an important connection between the theory of minimal surfaces and PDEs. In particular, the characterization of solutions of this equation has been a subject of intense interest in the past few decades. One of the most well known problems related to this subject is the *De Giorgi's conjecture* which states that the only bounded monotone solutions to Allen-Cahn equation in $\mathbb{R}^n$ are one-dimensional. Our main purpose in this talk is to characterize unstable solutions of least energy of the Allen-Cahn equation in the entire sphere $\mathbb{S}^n$. We prove that the ground state solutions are radially symmetric. Moreover they are unique up to rotations and correspond to the equator as a minimal hypersurface. Such information is useful in the study of the first critical values of a min-max sequence for the associated energy and a bifurcation problem.

Joint work with Pedro Gaspar, Marco Guaraco and Henrik Matthiesen.

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*e-mail: rayssacaju@gmail.com