Flows in elastic-walled Hele-Shaw cells: from understanding fundamental interfacial instabilities to designing a fluidic fuse

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Abstract

Flow between two rigid parallel plates separated by a small gap, or a Hele-Shaw cell, was first studied experimentally in 1898. It has been since widely used, for example, as a laboratory model for porous media and for design of microfluidics, and is equally appealing to both experimentalists and theoreticians. The former phrase its simplicity of design and amenability to experimental measurements. The later rejoice in the quasi-two-dimensionality of the fluid flows caused by the large aspect ratio in the cell geometry, which significantly simplifies the mathematics of the problem. In our talk we will study flows in variants of this classical set up - elastic-walled Hele-Shaw cells, in which one of the rigid plates is replaced by an elastic wall.

In the first half of the talk, we will discuss fingering instability at the interface of air and a viscous fluid, which readily develops in the classical Here-Shaw cells and is an archetypical problem of pattern formation in fluid mechanics. We find a surprisingly effective means of suppressing this instability by replacing one of the rigid plates with a thin elastic membrane[1]. The resulting fluid-structure interaction fundamentally alters the interfacial patterns that develop and considerably delays the onset of fingering. The novel setup provides a missing link between two classical interfacial instabilities, the viscous fingering and the printer's instability (see Figure 1 (a, c)).

In the second half of our talk, we focus on a compliant Hele-Shaw cell in which the bottom wall has been replaced by a soft confined elastomer[2]. In the presence of a flow, the cell behaves in a manner analogous to the electrical fuse; above a critical flux, the flow-induced deformation of the cell blocks the outflow, interrupting (choking) the flow. In particular, the pressure distribution within the fluid applies a spatially variant normal force to the soft boundary, which causes nonuniform deformation. As a consequence of lateral confinement and incompressibility of the soft material, this flow-induced elastic deformation manifests as bulging near the cell outflow; bulges that come into contact with the rigid cell roof interrupt the flow (see Figure 1 (b, d)). This paves the way for the integration of passive flow limiters (which remove the need for external actuation) into microfluidic devices.

[1] D. Pihler-puzović et al., *"Viscous fingering in a radial elastic-walled Hele-Shaw cell."*, J. Fluid Mech., **849**, 163–191, 2018.

[2] F. Box et al., *"Flow-induced choking of a compliant Hele-Shaw cell."* Proc. Nat. Ac. Sc., **117**, 48, 30228–3023, 2020.

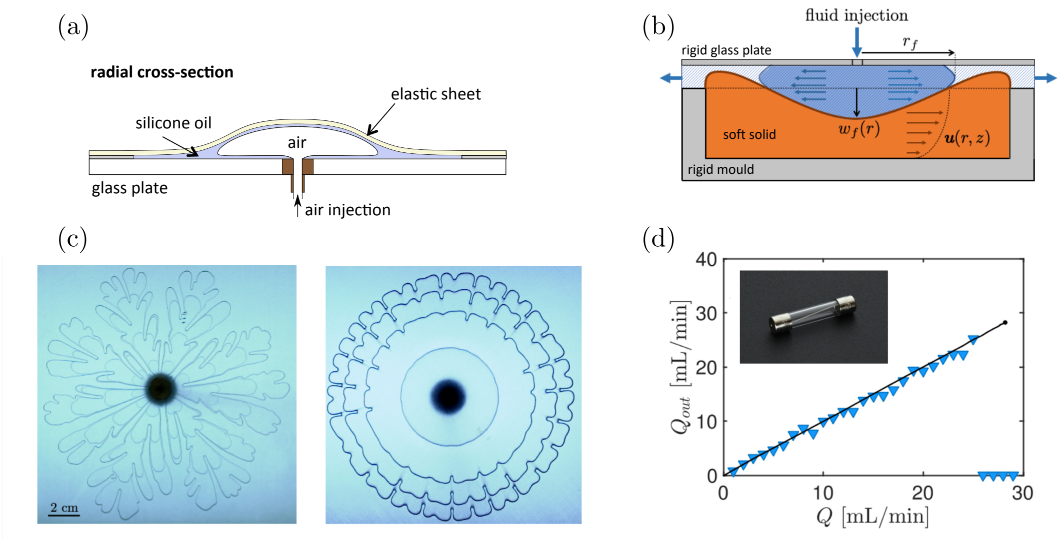

Figure 1: Schematic diagrams of a compliant Hele-Shaw cell (a) with a thin elastic membrane as the top boundary, and (b) a slab of soft confined solid as the bottom boundary. Injected fluid (air or a viscous liquid) spreads outwards, deforming the elastic boundary and displacing fluid resident in the cell. (c) The top view of the superimposed fingering patterns at the interface between two fluids when a viscous fluid is being displaced by an air bubble in a Hele-Shaw cell with rigid walls (left) and with an elastic wall from (a) (right). (d) The measured outflux Q_{out} as a function of the imposed volume flux Q of water injected into a small-scale Hele-Shaw cell from (b). For sufficiently high Q, the bulges near the cell rim make contact with the cell roof, blocking the outflow and interrupting the flow in a manner analogous to an electrical fuse.