

# XIII Summer Workshop in Mathematics

## Mechanics and Applied Mathematics Session

	Monday 08 Feb 21	Tuesday 09 Feb 21	Wednesday 10 Feb 21	Thursday 11 Feb 21	Friday 12 Feb 21
09:00 – 10:00				<b>Plenary Lecture (Mechanics) Maria Ekiel-Jeżewska</b>	
10:00 – 10:30					
10:30 – 12:30	<i>Short Talks</i> 10:30 – 11:00 Talk 1: <i>Santana</i> 11:00 – 11:30 Talk 2: <i>Matias</i> 11:30 – 12:00 Talk 3: <i>Sampaio</i> 12:00 – 12:30 Talk 4: <i>Rocha</i>	<i>Short Talks</i> 10:30 – 11:00 Talk 5: <i>Modesto</i> 11:00 – 11:30 Talk 6: <i>Bufo</i> 11:30 – 12:00 Talk 7: <i>Irilan</i> 12:00 – 12:30 Talk 8: <i>Gois</i>	<i>Short Talks</i> 10:30 – 11:00 Talk 9: <i>Sá</i> 11:00 – 11:30 Talk 10: <i>Oliveira</i> 11:30 – 12:00 Talk 11: <i>Oliveira</i> 12:00 – 12:30 Talk 12: <i>Póvoa</i>	<b>10:30 – 11:30 Draga Pihler-Puzović</b>  <b>11:30 – 12:30 Áine Byrne</b>	
12:30 – 14:30					
14:30 – 15:30					
15:30 – 16:00					
16:00 – 18:00	<i>Short-Course:</i>  <i>Level-Set Method: a hands on course.</i> Taygoara F. De Oliveira	<i>Short-Course:</i>  <i>Level-Set Method: a hands on course.</i> Taygoara F. De Oliveira	<i>Short-Course:</i>  <i>Level-Set Method: a hands on course.</i> Taygoara F. De Oliveira	<b>16:00 – 17:00 Aline de Souza Paula</b>  <b>17:00 – 18:00 Sarah Hormozi</b>	

Color key:

- 1 hour lectures given by invited guests.
- 30 min scientific talks.
- Mini-course (beginner level).

## SHORT COURSE

Title: Level Set mão na massa: uma introdução prática.

Author: Taygoara Felamingo de Oliveira

Abstract: O objetivo desse mini curso é fornecer uma introdução ao método de Level Set para a captura de interfaces. O método de Level Set pode ser considerado uma “tecnologia numérica” bem estabelecida, útil na simulação do movimento de interfaces como os observados no escoamento de gotas e bolhas, ondas, cápsulas e do sangue (na escala das hemácias). Também é empregado no caso de interfaces que são impelidas a moverem-se em razão de outros mecanismos. Por isso, encontra aplicações em processamento de imagens, computação gráfica, simulação de processos biológicos e até em astrofísica. Nesse curso, vamos estudar os elementos do método: equações básicas, discretização numérica, suavização de interfaces, técnicas de reinicialização e acoplamento com um escoamento externo. Vamos realizar implementações durante o curso para demonstrar as características da técnica. Haverá atividades práticas, desenvolvidas em Octave. Para melhor aproveitamento do curso, são desejáveis conhecimentos prévios de equações diferenciais, métodos numéricos básicos e de programação (só um pouquinho já é suficiente!).

## SCIENTIFIC SHORT TALKS

MONDAY, 08 FEBRUARY 2021

10:30 – 11:00 Talk

Title: Numerical simulation of the influence of the nonlinear effects on the active noise control

Authors: Guilherme Mendes Santana, Roberto Francisco Bobenrieth-Miserda

Abstract: High bypass ratio turbofans are the most used propulsion system in modern aircraft and are responsible for a significant part of the aircraft noise. The broadband noise is usually associated with the turbulent effects of the boundary layers and vorticity wakes. However, the results of research from the Computational Aeroacoustics Laboratory in the University of Brasilia show that there is at least a part of the broadband noise that cannot be explained by the boundary layer effects. Thus, the objective of this work is to investigate if the nonlinear effects associated with the large pressure fluctuations cause the appearance of a broadband component in the noise that results from the interaction of finite-amplitude sound waves. This work is composed of three parts, which consist of the study of different types of interactions between sound waves. The first part consists of the simulation of sound waves that interact inside an acoustic resonator and form a standing wave. The second consists of the simulation of the noise cancelation due to the interaction of two standing waves with opposing phases. The third part consists of the simulation of the continuous emission of sound waves into the resonator. These simulations use a moving-body immersed boundary method to solve the time-dependent, compressible, one-dimensional Euler equations. The domain discretization uses a finite volumes method, with fluxes calculated with fourth-order precision in space, and the time marching process consists of a third-order Runge-Kutta scheme. To properly simulate the waves generated by an oscillating piston, it was necessary to implement

a change in the immersed boundary method to capture the movement of the boundary inside the control volumes. The results showed that there is no change from tonal to broadband noise due to the nonlinear effects for frequencies within the human hearing range.

11:00 – 11:30 Talk 2

Title: Rivulet Flow Down a Vertical Surface: Lubrication Analysis and Numerical Solution

Authors: Gustavo Carreiro Matias, Rafaela Moreira Borges, André von Borries Lopes

Abstract: In this work, the flow of a rivulet of incompressible Newtonian fluid down a vertical plane is investigated. This problem has been extensively studied due to the large number of applications in which it can be observed. In most works found in the literature, the lubrication approximation is used to obtain a reduced model. We show that the widely used model is not the most appropriate for the solution of this problem. In addition, a new reduced model is developed with an appropriate perturbation parameter and the effect of the free surface curvature conservation in the lubrication analysis is also evaluated. It is shown that this new model considerably extends the application range of the lubrication analysis. To verify this statement, the full problem is solved using the finite element method. It is possible to obtain the velocity field of the flow for contact angles between  $0^\circ$  and  $180^\circ$ . Comparing the models presented with the complete solution, it becomes evident that incorporating the exact expression of the free surface shape in the reduced model further increases the application of the lubrication theory.

11:30 – 12:00 Talk 3

Title: Convergence Analysis for Explicit and Implicit Interpolation Methods in Finite Volumes

Authors: Luiza Sampaio, Roberto Francisco Bobenrieth-Miserda

Abstract: In this work, grid-convergence analyzes are performed for different interpolation methods, explicit and implicit, used in numerical simulations based on finite volume discretizations. The main differential of this work is in the approach given to the convergence analysis, which is performed in the domains of space and time together. The objective is to verify, for each of the interpolation methods, whether the convergence-order corresponds to the convergence-order of design and, with this information, determine the computational cost/benefit ratio for each of the explicit and implicit methods tested. For this, the shock-tube problem will be used, which will be simulated using the VAT code (Virtual Aeroacoustic Tunnel) developed by the Computational Aeroacoustics Laboratory of the University of Brasília (Caalab-UnB).

12:00 – 12:30 Talk 4

Title: Poiseuille flow inside ducts with rectangular cross section: analytical and numerical approach

Author: Wildemberg Ribeiro Rocha, André von Borries Lopes

Abstract: The modeling and study of how flows are developed inside rectangular ducts has a wide range in the mechanical industry as well as in the modeling of complex systems, so that the results obtained here can be implemented in the optimization of products and processes. The present study investigates a classic solution of the Navier-Stokes equations: the laminar flow of an incompressible Newtonian fluid in a rectangular duct. In order to

model and solve the problem analytically, we combine the solution for the plane Poiseuille flow (1844) and a Fourier series method. In addition, a numerical investigation of the problem is carried out through the Finite Differences Method (FDM), which is based on the solution of differential equations through the approximation of its derivatives by finite differences taken over an appropriate grid. Finally, a comparative study between the results obtained in the present work and those available in the literature is presented.

**TUESDAY, 09 FEBRUARY 2021**

10:30 – 11:00 Talk 5

Title: Aggregation patterns in systems composed of few magnetic particles

Authors: Jorge Augusto Cassis Modesto, Yuri Dumaresq Sobral

Abstract: In this work, we study the aggregation patterns that we observe in systems composed of two and three magnetic particles interacting magnetically and via solid-solid contact in two-dimensions. We use a discrete element method to take into account solid-solid interactions between pairs of particles. The particles are initially separated and, in the case of three particles, placed in the vertex of an equilateral triangle, and then are allowed to evolve for a given period of time. The final configurations are identified and associated to different levels of magnetic potential energy. We construct bifurcation diagrams for both cases by considering several initial conditions of the orientation of the magnetic dipole and identifying the final state configurations. In both cases, the majority of the final configurations is the aggregation of the particles in chains, but a non-negligible number of initial configurations leads to complete dispersion of the particles. Finally, we explore in more detail the surprising fractal nature of the bifurcation diagram.

11:00 – 11:30 Talk 6

Title: Collapsing columns in DEM: Differences between integration schemes.

Authors: Gabriel Nóbrega Bufolo, Yuri Dumaresq Sobral

Abstract: Choosing an integration method for a DEM simulation is a hard task, made harder by the fact that there is no correct choice. Each method has its advantages and knowing these tradeoffs and how they will alter what your software is capable of simulating as well as how it affects the results of the collapse. In this talk we present some of these tradeoffs, such as: computational performance, path convergence of individual particles, energy conservation, greatest time step for good collision resolution and impact on global measures (such as runout distance).

11:30 – 12:00 Talk 7

Title: A comprehensive study of locomotion of particles and microorganism at low Reynolds number

Authors: Yves Garnard Irilan, Francisco Ricardo Cunha

Abstract: This work presents an investigation on the locomotion of particles and organisms at low Reynolds numbers using mathematical, numerical models as well as experimental studies. First, we simulate an artificial swimmer to capture the physics involved, we carried out some experiments with prototype microorganisms in creeping flow motion for this end a macroscopic swimmer which propels itself by mimicking the helical flagella is developed. Three swimming models propelled by a helical tail with different wavelengths are tested and the movement is examined for the ambient solvent being Newtonian or non-Newtonian. Secondly, we apply the Slender Body Theory (SBT) in order to describe the dynamics of swimming micro-organism with motion produced by helical flagellum propulsion. Also the problem was modelled and simulated using computational fluid dynamics (CFD). The propulsive velocity, the trajectory, propulsive force, and torque are measured in the experiments and they are also compared with those predicted using numerical simulation as function nondimensional parameters Strouhal number and Deborah number. A very good agreement and compatibility were registered between the experimental and numerical results. Finally, we study active matter magnetic organisms. For this end, we perform simulation with a model of a kind of magnetotactic bacteria composed of a cluster of spherical magnetic nanoparticles aligned in chain. This enable this magnetic bacteria to respond even to weak magnetic fields such as the one of the Earth (about 0,7 Gauss). In our simulation the motion of the magnetic organisms is investigated in the presence of steady and oscillatory fields. Since the suspended magnetic organisms are easily manipulated and controlled under an applied field, the system studied here behaves like an active suspension. We verify collective response of the magnetic organisms in the presence of an external applied fields. In order to explore collective effects in the magnetic suspension investigated, hydrodynamic and dipolar particle interactions under condition of creeping flow are incorporated to the computer simulations by using appropriated Green's functions.

12:00 – 12:30 Talk 8

Title: A dinâmica de casais lineares: condições matemáticas para o sucesso de uma relação.

Authors: Alan Santos Gois, Yuri Dumaresq Sobral, Jorge Carlos Lucero, Andrea Genovese de Oliveira

Abstract: O primeiro modelo matemático que descreve a dinâmica de relações amorosas foi proposto por Steven Strogatz (Mathematics Magazine, 61, p.35, 1988) e tinha como base um sistema de equações diferenciais ordinárias lineares autônomas. De fato, considerando-se um casal típico (Eduardo e Mônica, por exemplo), podemos definir  $E(t)$  como o sentimento de Eduardo por Mônica no instante  $t$  e  $M(t)$  o sentimento de Mônica por Eduardo em  $t$ . Se  $E > 0$ , Eduardo está apaixonado por Mônica. Se  $E < 0$ , Eduardo odeia Mônica e se  $E = 0$ , Eduardo é indiferente a Mônica. Definições similares se aplicam para  $M(t)$ . O sistema proposto é linear e pode ser escrito da seguinte forma:  $dE/dt = aE + bM$ ;  $dM/dt = cE + dM$ , em que  $a$ ,  $b$ ,  $c$  e  $d$  são constantes. O único ponto fixo deste modelo está associado à indiferença mútua do casal, e a ausência de pontos fixos (estáveis) no primeiro quadrante do espaço de fase atrela o sucesso de uma relação a que  $E$ ,  $M$  tende à infinito quando  $t$  tender à infinito. Neste trabalho, mostramos quais são as condições para as constantes do modelo de tal modo que tenhamos esse sucesso. Neste trabalho, classificamos os perfis de casais em função das constantes e mostramos quais são as condições necessárias para obtermos o sucesso, bem como os casais que satisfazem tais condições.

**WEDNESDAY, 10 FEBRUARY 2021**

10:30 – 11:00 Talk 9

Title: Modelling of aqueous humour flow in the human eye

Authors: Caio Vinícius Schurgelies de Sá, André von Borries Lopes

Abstract: We present the fluid flow of aqueous humour in the anterior chamber of the human eye, discussing how mathematical and physical modelling can help to understand its behavior. The relative importance of buoyancy driven flow of aqueous humor is of particular interest. After a brief review of basic eye physiology, we introduce the anterior chamber model of Canning et al. (2002). Using classical fluid dynamics tools, such as the Boussinesq approximation, we present the governing equations of the flow, with proper boundary conditions. We employ the so-called lubrication theory to simplify the governing equations and solve them analytically. The velocity and temperature fields are analyzed for physical insight and understanding of the model limitations. We then introduce a numerical methodology, using the open-source software tools of SU2. The numerical solution is discussed and compared with the analytical approach. The two methodologies are in good agreement considering the general behavior of the flow, but the analytical solution doesn't capture all its features. We also conclude that the buoyancy driven flow is the dominant mechanism in the anterior chamber.

11:00 – 12:00 Talks 10 and 11

Title: Análise de Estabilidade de Modelos de Proliferação de Doenças Infecciosas

Authors: Carolina Barros de Oliveira (talk 10), Rafael Medeiros da Cunha Oliveira (talk 11), Andrea Genovese Oliveira

Abstract: Em meio ao cenário de contaminação por doenças infecciosas, o estudo de modelos matemáticos epidêmicos e endêmicos mostrou-se muito importante na descrição da forma de transmissão de doenças por agentes infecciosos. Por meio de simulações numéricas, análises quantitativas e qualitativas, o estudo dos modelos pode facilitar o entendimento de como as doenças infecciosas se espalham, ajudar a identificar parâmetros importantes e assim, tem-se a possibilidade de estipular o comportamento da doença e então providenciar políticas de prevenção e controle.

12:00 – 12:30 Talk 12

Título: Introdução à modelagem matemática do fluxo sanguíneo

Authors: Gabriel Silva Póvoa, Andrea Genovese Oliveira

Abstract: O fluxo sanguíneo no sistema arterial humano pode ser considerado um problema matemático de dinâmica de fluidos. A simulação do fluxo sanguíneo no sistema da rede arterial fornece uma melhor compreensão da fisiologia do corpo humano. A partir de uma revisão literária de modelos hemodinâmicos, este estudo apresenta uma modelagem matemática do fluxo sanguíneo arterial derivada das equações de Navier-Stokes e algumas suposições para simplificação do problema. A partir do sistema de equações diferenciais parciais não lineares para o fluxo sanguíneo e a área da seção transversal da artéria, foi adotado um método numérico para resolver as equações e analisar os resultados por meio de gráficos.