

On the Arithmetic Behaviour of Liouville Numbers Under Rational Maps

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Abstract

The genesis of transcendental number theory, took place in 1844 with Liouville's result "bad" approximation of algebraic numbers by rationals. More precisely, if α is an algebraic number of degree $n > 1$, then there exists a positive constant C , such that $|\alpha - p/q| > Cq^{-n}$, for all $p/q \in \mathbb{Q}^*$. Using this remarkable fact, he was able to build a non enumerable set of transcendental numbers called *Liouville numbers*. Since then, several classifications of transcendental numbers have been developed, one of them proposed by Kurt Mahler in 1932. He splitted the set of transcendental numbers on three disjoint sets: S -, T - and U -numbers. In a certain sense, U -numbers generalize the concept of Liouville numbers. Yet, the set of U -numbers can be splitted into U_m -numbers, that are numbers "rapidly" approximable by algebraic numbers of degree m . On this lecture, the following result, made in cooperation with D. Marques and P. Trojovský, will be proved:

Theorem: Let $\omega : \mathbb{N} \rightarrow \mathbb{N}$, such that $\omega_n \rightarrow \infty$, as $n \rightarrow \infty$. Let $\xi \in \mathbb{R}$ be a Liouville number, such that there exists an infinite sequence of rational numbers $(p_n/q_n)_n$, satisfying

$$\left| \xi - \frac{p_n}{q_n} \right| < H \left(\frac{p_n}{q_n} \right)^{-\omega_n},$$

where $H(p_{n+1}/q_{n+1}) \leq H(p_n/q_n)^{O(\omega_n)}$. Now, take $\alpha_0, \dots, \alpha_l, \beta_0, \dots, \beta_r \in \overline{\mathbb{Q}}$, with $\beta_r = 1$ and $\alpha_l \neq 0$, such that $[\mathbb{Q}(\alpha_0, \dots, \alpha_l, \beta_0, \dots, \beta_r) : \mathbb{Q}] = m$. Then, for $P(z), Q(z) \in \overline{\mathbb{Q}}[z]$, given by $P(z) = \alpha_0 + \alpha_1 z + \dots + \alpha_l z^l$ and $Q(z) = \beta_0 + \beta_1 z + \dots + \beta_r z^r$, $P(\xi)/Q(\xi)$ is a U_m -number.

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