

Counterexamples to a Conjecture of Norton

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Abstract

Let $\Gamma^*(k)$ be the smallest integer s such that the equation

$$a_1x_1^k + \cdots + a_sx_s^k = 0$$

has a nontrivial solution in every p -adic field \mathbb{Q}_p , regardless of the values of the (rational integer) coefficients. An old conjecture of Norton was that we should have $\Gamma^*(k) \equiv 1 \pmod{k}$ for all degrees k . This was disproved in 1974 by Bovey, who showed that $\Gamma^*(8) = 39$, but until a few years ago this was the only known counterexample. In this talk, we show that there are infinitely many counterexamples to Norton's conjecture.

References

- [1] M. P. Knapp and H. Godinho, Infinitely many counterexamples to a conjecture of Norton, *Michigan Math Journal*, 2020, 533-543.