

# A surprising formula for Sobolev norms

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## Abstract

The Sobolev spaces, introduced in the 1930s, have become ubiquitous in Analysis and Applied Mathematics. They involve  $L_p$  norms of the gradient of a function  $f$ . We present an alternative point of view where derivatives are replaced by appropriate finite differences and the Lebesgue space  $L_p$  is replaced by the slightly larger Marcinkiewicz space  $M_p$  (aka weak  $L_p$  space). Surprisingly, these new spaces coincide with the standard Sobolev spaces, a fact which sheds a new light onto these classical objects and should have numerous applications. In particular, it rectifies some well-known irregularities occurring in the theory of fractional Sobolev spaces. The proof relies on original calculus inequalities which might be useful in other situations. The lecture is based on a recent joint work with Jean Van Schaftingen and Po-Lam Yung.