

# Bifurcations in Random Dynamical Systems

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## Abstract

In this talk, an overview of recent progress on bifurcations in random dynamical systems [1] will be given, and the focus will be on a detailed analysis of the pitchfork bifurcation under additive noise. It is well known that under addition of noise, certain properties of the pitchfork bifurcation are destroyed [2]. I would like to present three different points of view that demonstrate that the bifurcation is still present. The first two approaches concern a finite-time and local analysis of the (unbounded noise) random dynamical system, revealing structural changes at the bifurcation point that can be described by the dichotomy spectrum and conditional Lyapunov exponents [3, 4]. In the last part of the talk, we study the situation for bounded noise, and show that the pitchfork bifurcation can be described by means of a discontinuous bifurcation of minimal invariant sets. Joint work with Mark Callaway, Thai Son Doan, Maximilian Engel, Jeroen Lamb, and Christian Rodrigues.

## References

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