Lindelöf Spaces and Michael's Problem*

Alexssandra Thais Pereira Alves de Souza †
Universidade Federal do ABC (UFABC)
Santo André, Brasil

Abstract

Lindelöf spaces are a natural generalization of compact spaces, which present several differences compared to compactness in their basic properties. For instance, unlike compactness, Lindelöfness is not preserved by the product topology. Particularly, we can ask whether the topological product of a Lindelöf space and the set of irrational numbers must be a Lindelöf space. That question is known as Michael's problem, and is one of the principal open problem in general topology. A regular Lindelöf space such that its product topology with the set of irrationals is Lindelöf is called a Michael space; some consistent examples of Michael spaces exist but a final answer for Michael's problem is yet to be obtained.

The main goal of this project is to study Lindelöf spaces and some of the principal open problems related to them. We study the Arkhangel'skiĭ theorem about the cardinality of first countable Lindelöf spaces and the Hajnal and Juhász problem about the reflection of the Lindelöf property in subspaces of cardinality $\aleph_1$.

References

[1] A. V. Arhangel'skii, *On the cardinality of bicompacta satisfying the first axiom of countability*, Soviet Mathematics. Doklady **10** (1969), 951–955.

[2] P. S. Alexandroff e P. S. Urysohn, *Mémoire sur les espaces topologiques compacts*, Proceedings of the Koninklijke Nederlandse Akademie van Wetenschappen. Series A **14** (1929), 1–96.

[3] A. Hajnal e I. Juhász, *Remarks on the cardinality of compact spaces and their Lindelöf subspaces*, Proceedings of the American Mathematical Society **59**:1 (1976), 146–148.

[4] E. A. Michael, *Paracompactness and the Lindelöf property in finite and countable Cartesian products*, Compositio Mathematica **23**:2 (1971), 199–214.

[5] E. Michael, *The product of a normal space and a metric space need not be normal*, Bulletin of the American Mathematical Society **69** (1963), 375–376.

*Supported by FAPESP, process number 19/11642-0.

†e-mail: alexssandra.thais@aluno.ufabc.edu.br