Unique solutions for functional Volterra–Stieltjes integral equations

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Abstract

In this work, we deal with integral functional Volterra–Stieltjes integral equations:

$$\begin{cases} x(t) = \phi(0) + \int_{t_0}^{t} a(t,s) f(x_s, s)\mathrm{d}g(s), & t \geq t_0, \\ x_{t_0} = \phi, \end{cases} \tag{1}$$

where $t_0 \in \mathbb{R}$, $\phi \in G([-r,0],\mathbb{R}^n)$, $f : G([-r,0],\mathbb{R}^n) \times [t_0,+\infty) \to \mathbb{R}^n$, $a : [t_0,+\infty)^2 \to \mathbb{R}$, $g : [t_0,+\infty) \to \mathbb{R}$, $x_s : [-r,0] \to \mathbb{R}^n$ is defined by $x_s(\theta) = x(s+\theta)$ and the integral on the right–hand side is in the sense of Henstock–Kurzweil–Stieltjes.

We shall present some conditions with respect to the functions a and g, as well as some conditions with respect to the integral

$$\int_{\tau_1}^{\tau_2} b(t,s) f(x_s, s)\mathrm{d}g(s),$$

when $b : [t_0,+\infty)^2 \to \mathbb{R}$ is a regulated function and $t_0 \leq \tau_1 \leq \tau_2 \leq t_0 + \sigma < d$, for some $0 < \sigma < d - t_0$.

With these conditions, we will be able to prove the existence and uniqueness of local and maximal solutions for equation (1).

References

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